THE ORDER OF NUMBERS AND THE COLLATZ PROBLEM

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ABSTRACT

The algorithm of Collatz is also known the 3n+1 problem. It will be started with any uneven natural number (n). This number is to triple and add one. Then divide by two as often as possible, if the number is even. If the number is odd, use again the 3n+1 step. The conjecture of the Collatz problem is that these operations always reach 1, no matter which positive integer is chosen to start the algorithm. A view to an order of numbers gives an explanation for the dominance of degressive steps to reach 1. The idea to identify the order of numbers is developed from an interview of Dr. Peter Plichta where he spoke on his book "Das Primzahlenkreuz".

KEYWORDS

Number theory, Order of integer numbers, Collatz problem

1. INTRODUCTION

The Collatz problem is given with an algorithm of degressive and progressive steps. Started on a random uneven number the steps will be used. For the arbitrary uneven number (n) is to use: 3n+1 and the result can be divide by 2 as much as possible. If this divided number heads to an uneven number again, then also the step: 3n+1 is necessary. The algorithm ends at number 1 and can be continued indefinitely at the lowest level with 4,2,1.

2. SCHEME OF ORDER OF NATURAL NUMBERS

2.1. Order

Based on a simple partition technique all the two and three divisible numbers appear in separate columns. This ascending order allows endless generation or identification of affiliation to the three columns or classes. First natural numbers are separated in ascending fashion in three columns with given rule. The natural numbers 1, 2 and 3 are the header for three main-columns. To build up, it is useful to start with the column 3 where all three-divisible numbers are placed. In column 2, all two-divisible numbers are found. The successive numbers can be separate endless adequate to the properties and allows another view to the structure of the natural and also integer numbers.

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Table 1. Cutout of separation natural numbers at three columns with 1, 2 and 3 as head.

1	**2**	**3**
5	4	6
7	8	9
11	10	12
13	14	15
17	16	18
…	…	…

2.2. Relevance of order of numbers to the Collatz problem

The showed order of numbers, see Table 1 is reflected first an equipartition of numbers in three columns. For the Collatz problem are to consider only the columns 2 and 3. With the multiplying of uneven numbers to factor 3, all numbers redraft to column 3 (3n). With addition of 1, they is changed to an even number and can be divided by 2 as much as possible. The progress of divide has to be in dependence of recurred even numbers underneath. The view of columns allowed to see a degree of frequency for the two divisible numbers. Every second number in column 3 is also even. The degressive probability (n/2) has a ratio of 3 to the progressive (3n+1).This is the reason of the convergent to the number 1.

$$\{2n,\ 3n \in 2n\} = 3\{3n \notin 2n\} \qquad n \in N$$

$$\text{or} \quad |A \cup (A \cap B)| = 3|B \setminus (A \cap B)| \qquad A=\{n \in N | 2n \in N\},\ B=\{n \in N | 3n \in N\}$$

2.3. Open Questions

In some cases a lot of steps are necessary to achieve the forecast result, e.g. for number 31, 47, 63and it seems, that the Collatz algorithm diverge infinite and maintain again and again to higher numbers, who are often prime numbers. Is it possible to predict, which numbers maintain to a high divergence before it convergence against 1? There is a noticeable symmetry in total stopping time of numbers. Which property of numbers is to recognize in this symmetry?

3. CONCLUSIONS

The separation approach of all natural numbers in three classes with different properties allows identifying the frequency of properties between the classes of numbers. Two classes, Column 2 and 3 are relevant for the Collatz problem. With representation of numbers in this classes an overview about the even numbers is given. The Collatz algorithm combines degressive steps - divide by two -, with progressive steps - multiply by 3 and add 1-, which guide the Algorithm of Collatz always between columns 2 and 3. The part of even numbers is simple to check in the order of numbers and has a rate of 3.This explains the higher degressive tendency compared to progressive tendency of the Collatz algorithm. The convergence at number 1 and there endless alternating about them is unavoidable.

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REFERENCES

[1] Jacqueline Wötzel, (2023) Order of integer numbers and structure of prime numbers, Inter-National Journal on Computational Science & Applications (ILJSCA) Vol.13, No.1, pp1-2

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