A COMPARATIVE PERFORMANCE STUDY OF OFDM SYSTEM WITH THE IMPLEMENTATION OF COMB PILOT-BASED MMSE CHANNEL ESTIMATION

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ABSTRACT

This paper presents a comparative performance analysis of wireless orthogonal frequency division multiplexing (OFDM) system with the implementation of comb type pilot-based channel estimation algorithm over frequency selective multi-path fading channels. The Minimum Mean Square Error (MMSE) method is used for the estimation of channel at pilot frequencies. For the estimation of channel at data frequencies different interpolation techniques such as low-pass, linear, and second order interpolation are employed. The OFDM system simulation has been carried out with Matlab and the performance is analyzed in terms of bit error rate (BER) for various signal mapping (BPSK, QPSK, 4QAM, 16QAM, and 64QAM) and channel (Rayleigh and Rician) conditions. The impact of selecting number of channel taps on the BER performance is also investigated.

KEYWORDS

OFDM, Multipath fading channels, Comb Pilot, MMSE estimation, Interpolation, Signal mapping.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a parallel transmission technique and is widely used in wireless communication systems because of its high rate transmission capability and robustness against multipath fading, high spectral efficiency and so on [1]. However, as the radio channel is frequency selective and time-variant, the channel transfer function in OFDM systems looks unequal in both the time and frequency domains. Thus a dynamic estimation of the channel is necessary before demodulating the OFDM signals for the coherent detection of the information symbols.

Two basic training-based one dimensional (1D) channel estimation techniques can be adopted in OFDM system are block type and comb type pilot-based channel estimation. In block type pilot arrangement, the pilot signal is allocated to particular OFDM block and sent periodically in time domain. Instead, in comb type pilot arrangement, the pilot signals are uniformly distributed within each OFDM block. The comb pilot-based channel estimation consists of algorithms to estimate the channel at pilot frequencies and to interpolate the channel at data frequencies. The estimation of channel at pilot frequencies can be based on least square (LS), minimum mean square error (MMSE) or least mean square (LMS) method, while different interpolation techniques can be incorporated for the channel estimation at data frequencies [2, 3].

The aim of this paper is to evaluate the bit error rate (BER) performance of OFDM communication system with the implementation of comb type pilot-based MMSE channel estimation.
estimation algorithms and to compare the system performance for signal mapping with different digital modulation schemes over Rayleigh and Rician multi-path fading channels. In addition, the impact of selecting number of channel taps and different interpolation techniques such as low-pass, linear, and second order interpolation on the system performance are also examined. The organization of the paper is as follows. Relevant studies of pilot-based channel estimation in OFDM systems are discussed in Section 2. Section 3 describes a model of OFDM system used for the implementation. Comb type pilot-based channel estimation algorithm is briefly explained in Section 4. In section 5, simulation results are displayed in graphical form, analyzed and compared. Conclusions of the present work are given in Section 6.

2. RELATED WORKS

In earlier studies the performance of different pilot-based channel estimation techniques both in the time and frequency domains for wireless OFDM communication systems were investigated [4-6]. The authors in [7] reviewed block type and comb type pilot-based channel estimation and revealed that comb-type pilot based channel estimation with low-pass interpolation performs the best among all channel estimation algorithms. Cai and Giannakis [8] studied the OFDM system performance with M-PSK digital modulation over Raleigh frequency fading channel and optimized the number of pilot symbols, the placement of pilot symbols and the power allocation between pilot and information symbols.

In [9-11], OFDM channel estimation with MMSE estimator was investigated. The authors in [10] shown that among the MMSE and ZF equalizers, the BER performance of MIMO OFDM system is better for MMSE equalizer. Pramano and Triyono [12] found that the performance of MIMO system is better than SISO system and the MMSE estimator produced better performance than low complexity LS estimator. In another study [13], it was concluded that block type pilot-based channel estimation is better for slow fading channel while comb type pilot-based channel estimation shows better performance in fast fading environment.

3. OFDM SYSTEM DESCRIPTION

Figure 1 shows a typical block diagram of OFDM communication system with comb type pilot-based MMSE channel estimation. The binary data (information) from the input are first grouped and mapped into multi-amplitude and multi-phase signals according to the type of modulation (BPSK, QPSK, 4QAM, 16QAM, and 64QAM) used at the signal mapper. After serial to parallel conversion of the modulated data, comb type pilots are inserted uniformly between the information data sequence. The IFFT block is used for transforming and multiplexing the complex data sequence into time domain signal. Following the IFFT block, a cyclic prefix is added to avoid possible inter-symbol interference (ISI) in OFDM systems. After the parallel to serial conversion, the signal is transmitted through a frequency selective multi-path Rayleigh or Rician fading channel.

At the receiver, after serial to parallel conversion of the received signal, the cyclic prefix is removed first and then fed to an FFT block for de-multiplexing the multi-carrier signals. Following the FFT block, the pilot signals are extracted from the demultiplexed samples. The channel estimation at pilot subcarriers is performed by MMSE while the estimation of channel at data subcarriers is achieved by different interpolation techniques. After signal demodulation at the signal demapper, the information binary data is reconstructed at the receiver output.

4. COMB PILOT-BASED CHANNEL ESTIMATION

Comb type pilot-based channel estimation is suitable for fast-fading channel where the channel condition changes between adjacent OFDM symbols. The comb type pilot arrangement is shown in Fig. 2 in which the pilot signals are uniformly distributed within each OFDM block. In comb
type channel estimation, after extracting the pilot signals from the received signal, the channel transfer function is estimated from the received pilot signals and the known pilot signals. The channel responses of data subcarriers can be estimated with the interpolation of the neighboring pilot channel responses.

![OFDM system block diagram with Comb pilot-based MMSE channel estimation.](image)

Comb type pilot-based channel estimation can be based on least square (LS), minimum mean square error (MMSE) or least mean square (LMS) method. Here only MMSE channel estimator is employed for the estimation of channel at pilot subcarriers because of its superior performance as compared to LS estimator. Pilot signal estimation and channel interpolation algorithms are discussed in the following subsections.
4.1 Channel estimation at Comb pilot subcarriers by MMSE

Suppose \( N_{pi} \) comb pilot signals \( X_{pi}(n) \), where \( n = 0,1,\ldots,N_{pi}-1 \), are uniformly added into \( X(m) \) data signals. If the OFDM signal modulated on the \( m \)-th subcarrier, \( X(m) \) can be expressed as

\[
X(m) = X(nL_s + i) = \begin{cases} 
X_{pi}(n), & i = 0 \\
\text{Information data}, & i = 1,2,\ldots,L_s - 1
\end{cases}
\]  

(1)

Here the total \( N \) subcarriers are divided into \( N_{pi} \) groups, each with \( L_s = N / N_{pi} \) adjacent subcarriers. The least squares (LS) estimate of pilot signal is given by [3, 4]

\[
\hat{H}_{pi,LS} = X_{pi}^{-1}Y_{pi}
\]

(2)

where

\[
H_{pi} = [H_{pi}(0)H_{pi}(1)\ldots\ldots H_{pi}(N_{pi}-1)]^T
\]

(3)

is the channel frequency response at pilot subcarriers,

\[
Y_{pi} = [Y_{pi}(0)Y_{pi}(1)\ldots\ldots Y_{pi}(N_{pi}-1)]^T
\]

(4)

is the vector of the received pilot signals. It can also be expressed as

\[
Y_{pi} = X_{pi}H_{pi} + I_{pi} + W_{pi}
\]

(5)

where

\[
X_{pi} = \begin{bmatrix} 
X_{pi}(0) & 0 \\
\vdots & \ddots \\
0 & X_{pi}(N_{pi}-1)
\end{bmatrix}
\]

\( I_{pi} \) is the ICI vector and \( W_{pi} \) is the Gaussian noise vector at pilot subcarriers.

The \( \hat{H}_{pi,LS} \) can also be expressed as

\[
\hat{H}_{pi,LS} = \left[ R_{H_{pi}H_{pi,LS}}^{-1} R_{H_{pi}H_{pi,LS}} \right] \hat{H}_{pi,LS}
\]

(6)

The estimate of pilot signals based on minimum mean square error (MMSE) is given by [4],

\[
\hat{H}_{pi,MMSE} = R_{H_{pi}H_{pi,pi}} R_{H_{pi}H_{pi,pi}}^{-1} \hat{H}_{pi,LS}
\]

\[
= R_{H_{pi}H_{pi}} \left( R_{H_{pi}H_{pi}} + \sigma_n^2 (X_{pi}X_{pi}^H)^{-1} \right)^{-1} \hat{H}_{pi,LS},
\]

where \( \hat{H}_{pi,LS} \) is the LS estimate of \( H_{pi} \), \( \sigma_n^2 \) is the noise variance of \( W_{pi} \), and the covariance matrices are defined by

\[
R_{H_{pi}H_{pi}} = E(H_{pi}H_{pi}^H), \quad R_{H_{pi}H_{pi,LS}} = E(H_{pi}H_{pi,LS}^H), \quad \text{and}
\]

\[
R_{H_{pi,LS}H_{pi,LS}} = E(H_{pi,LS}H_{pi,LS}^H).
\]
4.2 Channel estimation at data subcarriers by Interpolation

A highly efficient interpolation technique is necessary for the estimation of channel at data subcarriers using the channel information at pilot subcarriers. Three interpolation techniques: low-pass, linear and second order interpolations are employed for the present study. In low-pass interpolation, at first zeros are added to the original sequence. Then a low-pass finite impulse response (FIR) filter is applied to pass the original data sequence without any change and to interpolate between such that it can minimize the mean square error (MSE) between the interpolated points and their ideal values.

In linear interpolation, two successive pilot subcarriers that are located in between the pilots are used to determine the channel response for data subcarriers. It is implemented with the usage of digital filtering technique. The estimated channel response for data subcarriers \( m \),

\[
\hat{H}(m) = \hat{H}(nL_s + i)
\]  

The second order interpolation is implemented by a line time-invariant FIR filter. The estimated channel response is expressed by

\[
\hat{H}(m) = \hat{H}(nL_s + i)
\]  

\[
= D_1 \hat{H}_{pi}(n-1) + D_0 \hat{H}_{pi}(n) + D_{-1} \hat{H}_{pi}(n+1)
\]

where

\[
D_1 = \frac{\beta (\beta + 1)}{2}, \quad D_0 = -(\beta - 1)(\beta + 1), \quad D_{-1} = \frac{\beta (\beta - 1)}{2}, \quad \beta = i/N.
\]

5. SIMULATION RESULTS AND DISCUSSION

The simulation of OFDM system in Fig. 1 has been performed using Matlab 7.5 programming language. The parameters used for the simulation are listed in Table 1. The graphical representation of simulation results are shown in Figures 3 to 7. The Doppler frequency and number of channel taps are considered as 80 Hz and 20, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sub-carrier</td>
<td>256</td>
</tr>
<tr>
<td>Pilot Ratio</td>
<td>1/8</td>
</tr>
<tr>
<td>Pilot-to-Data Power Ratio</td>
<td>2</td>
</tr>
<tr>
<td>IFFT, FFT Size</td>
<td>256</td>
</tr>
<tr>
<td>Guard Type</td>
<td>Cyclic Extension</td>
</tr>
<tr>
<td>Cyclic Prefix (C.P.) Length</td>
<td>32</td>
</tr>
<tr>
<td>Constellation</td>
<td>BPSK, QPSK, 4QAM, 16QAM</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Rayleigh, Rician</td>
</tr>
<tr>
<td>Number of Channel Taps</td>
<td>20</td>
</tr>
<tr>
<td>Doppler Frequency</td>
<td>80 Hz</td>
</tr>
</tbody>
</table>

The BER performance of the simulated wireless OFDM system over Rayleigh multipath fading channel without and with the employment of comb type pilot-based channel estimation (LS and MMSE) is shown in Fig. 3. The input binary data were BPSK modulated and the channel
estimation at data subcarriers was obtained by low-pass interpolation. It is seen that without using any channel estimation algorithm, the bit error rate at the receiver is very high. However, with the employment of MMSE channel estimation, the OFDM system performance has greatly improved which results from the decrease of the amplitude and phase distortion of the transmitted signal because of multipath fading. In Fig. 3, comb pilot-based, low complexity, LS channel estimation is also incorporated for the comparison. It is clear that the MMSE channel estimator gives better BER performance than the LS estimator in data transmission over the Rayleigh channel. Similar results were observed in [12, 14] in studying the performance of channel estimation methods for SISO and MIMO OFDM system.

![Fig. 3 BER performance of OFDM system without and with comb pilot-based channel estimation (LS and MMSE).](image)

![Fig. 4 Performance of different interpolation techniques used in Comb pilot-based channel estimation of OFDM system](image)

Fig. 4 depicts the impact of various channel interpolation techniques on the performance of OFDM system. The simulation is performed under BPSK signal mapping over Rayleigh fading.
The performance of different interpolation techniques used in comb pilot-based MMSE channel estimation follows the sequence of best to the worst as low-pass, second order and linear. This result is consistent with earlier studies [9, 13].

The BER comparison of OFDM system with MMSE-interpolation-based comb type channel estimation for different number of channel taps is shown in Fig. 5. Low-pass interpolation and BPSK modulation were used for the system simulation over Rayleigh fading channel. The result shows that the system performance is increased when we choose the number of channel taps per multipath less than the CP length, whereas the performance getting worse if the number of channel taps exceed the CP length. As we increase the number of taps, the transmitted signal go under high degradation because of the increase of the number of times it would be reflected by the multipath channel taps and, thus, the system performance degrades [15].

Fig. 6 demonstrates the BER performance of OFDM system with comb pilot-based MMSE channel estimation over Rayleigh fading channel for BPSK, QPSK, 4QAM, 16QAM and 64QAM signal mapping. Low-pass interpolation was used to estimate channel frequency response (CFR) at data frequencies. Number of channel taps was chosen as 20. As seen the OFDM system outperforms at BPSK modulation [10, 11] and the performance degrades with increasing the order of the modulation over Rayleigh channel. The worst performance is observed at 64QAM modulation. For a typical SNR of 8 dB, the BER values for BPSK and 64-QAM modulations are 0.032 and 1.141, respectively. Accordingly, the system performance is improved by 15.5 dB with the use of BPSK modulation.

The BER performance of OFDM system with comb pilot-based MMSE channel estimation over Rician fading channel for BPSK, QPSK, 4QAM, 16QAM and 64QAM digital modulations are shown in Fig. 7. Channel taps was selected as 20 per multipath and low-pass interpolation was used for the estimation of channel at data frequencies. It is evident that the system provides better error rate performance with BPSK [10] than other modulations used over Rician channel. The OFDM system performance with BPSK is improved by 6.9 dB as compared to 64QAM at the SNR of 8 dB.
Comparing the BER performance of OFDM system with comb pilot-based MMSE channel estimation over Rayleigh and Rician multipath fading channels in Figs. 6 and 7, respectively, it is observable that the system performance over Rician channel is somewhat better than that over the Rayleigh channel environment [16]. For a typical SNR of 10 dB, the BER values for BPSK modulation over Rayleigh and Rician channels are 0.032 and 0.0165, respectively, which indicates that the system performance in Rician fading channel is improved by 2.88 dB.
6. CONCLUSIONS

In this paper, the BER performance of wireless OFDM system with the implementation of comb pilot-based channel estimation is examined for different signal mapping over multi-path Rayleigh and Rician fading channels. In channel estimation, the MMSE method is used for the estimation of channel at pilot subcarriers while different interpolation techniques are employed to estimate the channel at data subcarriers. From the present simulation based comparative study it can be concluded that the deployment of BPSK modulated OFDM system with MMSE and low-pass interpolation-based comb type channel estimation achieves good error rate performance in data transmission over all multipath fading environments involved.

REFERENCES