AN APPROACH TO DECREASE DIMENSIONS OF DRIFT HETERO-BOPOLAR TRANSISTORS

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ABSTRACT

In this paper based on recently introduced approach we formulated some recommendations to optimize manufacture drift bipolar transistor to decrease their dimensions and to decrease local overheats during functioning. The approach based on manufacture a heterostructure, doping required parts of the heterostructure by dopant diffusion or by ion implantation and optimization of annealing of dopant and/or radiation defects. The optimization gives us possibility to increase homogeneity of distributions of concentrations of dopants in emitter and collector and specific inhomogenous of concentration of dopant in base and at the same time to increase sharpness of p-n-junctions, which have been manufactured framework the transistor. We obtain dependences of optimal annealing time on several parameters. We also introduced an analytical approach to model nonlinear physical processes (such as mass- and heat transport) in inhomogenous media with time-varying parameters.

KEYWORDS

Drift heterobipolar transistor, analytical approach to model technological process, decreasing of dimensions of transistor

1. INTRODUCTION

In the present time performance of elements of integrated circuits (p-n-junctions, field-effect and bipolar transistors, ...) and their discrete analogs are intensively increasing [1-14]. To solve the problem they are using several ways. One of them is manufacturing new materials with higher speed of charge carriers [1-18]. Another way to increase the performance is elaboration of new technological processes or modification of existing one [1-14, 19, 20]. In this paper we introduce one of approaches of modification of technological to increase performance of bipolar transistor.

To solve our aim we consider hetero structure, which consist of a substrate and three epitaxial layers (see Fig. 1). One section have been manufactured in every epitaxial layer by using another materials so as it is presented on Fig. 1. After manufacturing of the section in the first epitaxial layer the section has been doped by diffusion or ion implantation to produce required type of conductivity (p or n) in the section. Farther we consider annealing of dopant and/or radiation de-
ffects. After that we consider manufacturing of the second and the third epitaxial layers, which also including into itself one section in each new epitaxial layer. The sections are also been manufactured by using another materials. Both new sections have been doped by diffusion or ion implantation to produce required type of conductivity ($p$ or $n$) in the sections. Farther we consider microwave annealing of dopant and/or radiation defects. Main aim of the paper is analysis of dopant and radiation defects in the considered heterostructure.

Fig. 1. Heterostructure, which consist of a substrate and three epitaxial layers with sections, manufactured by using another materials. View from side

2. Method of solution

To solve our aims we determine spatio-temporal distribution of concentration of dopant. We determine the required distribution by solving the second Fick’s law [1,3-5]

$$\frac{\partial C(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_c \frac{\partial C(x,y,z,t)}{\partial z} \right] + \frac{\partial C(x,y,z,t)}{T} = 0,$$

with boundary and initial conditions

$$\frac{\partial C(x,y,z,t)}{\partial x} \bigg|_{x=0} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial x} \bigg|_{x=L_x} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial y} \bigg|_{y=0} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial y} \bigg|_{y=L_y} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial z} \bigg|_{z=0} = 0, \quad \frac{\partial C(x,y,z,t)}{\partial z} \bigg|_{z=L_z} = 0, \quad C(x,y,z,0) = f(x,y,z).$$

Here $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant, $T$ is the temperature of annealing, $D_c$ is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials of the considered hetero structure, speed of heating and cooling of hetero structure (with account Arrhenius law). Dependences of dopant diffusion coefficient could be approximated by the following relation [3,21]
\[ D_c = D_e(x, y, z, T) \left[ 1 + \frac{C'(x, y, z, t)}{P'(x, y, z, T)} \right]^{1 + \frac{1}{\xi_1} \frac{V(x, y, z, t)}{V'} + \frac{1}{\xi_2} \frac{V^2(x, y, z, t)}{\left(V'\right)^2}}. \] (3)

where \( D_e(x, y, z, T) \) is the spatial (due to inhomogeneity of hetero structure) and temperature (due to Arrhenius law) dependences of diffusion coefficient; \( P(x, y, z, T) \) is the limit of solubility of dopant; value of parameter \( \gamma \) depends on materials of heterostructure and could be integer in the following interval \( \gamma \in [1,3] \) [3]; \( V(x, y, z, t) \) is the spatio-temporal distribution of concentration of vacancies; \( V' \) is the equilibrium distribution of concentration of vacancies. Concentrational dependence of dopant diffusion coefficient has been discussed in details in [3]. It should be noted, that using diffusion type of doping and radiation damage is absent in the case (i.e. \( \xi_1 = \xi_2 = 0 \)). We determine spatio-temporal distributions of concentrations of point radiation defects by solving of the following system of equations [21,22]

\[
\begin{align*}
\frac{\partial I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \\
&+ \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{i,v}(x, y, z, T)I(x, y, z, t)V(x, y, z, t) - k_{i,v}(x, y, z, T)V^2(x, y, z, t)
\end{align*}
\] (4)

\[
\begin{align*}
\frac{\partial V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \\
&+ \frac{\partial}{\partial z} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{i,v}(x, y, z, T)V(x, y, z, t) - k_{i,v}(x, y, z, T)V^2(x, y, z, t)
\end{align*}
\]

with initial

\[ \rho(x, y, z, 0) = f_\rho(x, y, z) \] (5a)

and boundary conditions

\[
\begin{align*}
\frac{\partial \rho(x, y, z, t)}{\partial x} \bigg|_{x=0}, \frac{\partial \rho(x, y, z, t)}{\partial x} \bigg|_{x=L_x} &= 0, \quad \frac{\partial \rho(x, y, z, t)}{\partial y} \bigg|_{y=0}, \frac{\partial \rho(x, y, z, t)}{\partial y} \bigg|_{y=L_y} = 0, \\
\frac{\partial \rho(x, y, z, t)}{\partial z} \bigg|_{z=0}, \frac{\partial \rho(x, y, z, t)}{\partial z} \bigg|_{z=L_z} &= 0.
\end{align*}
\] (5b)

Here \( \rho = I, V ; I(x, y, z, t) \) is the spatio-temporal distribution of concentrations of interstitials; \( D_I(x, y, z, T) \) are the diffusion coefficients of interstitials and vacancies; terms \( V^2(x, y, z, t) \) and \( V^2(x, y, z, t) \) correspond to generation of divacancies and diinterstitials; \( k_{i,v}(x, y, z, T) \), \( k_{i,v}(x, y, z, T) \) and \( k_{i,v}(x, y, z, T) \) are the parameters of recombination of point radiation defects and generation appropriate their complexes, respectively.

We determine spatio-temporal distributions of concentrations of divacancies \( \Phi_v(x, y, z, t) \) and diinterstitials \( \Phi_i(x, y, z, t) \) by solving the following system of equations [21,22]
temperature interval could be approximated by the following function

\[ k(x, y, z, T) = k_0 \exp \left( \frac{T - T_d}{T_d} \right) \]

where \( k(x, y, z, T) \) is the heat conduction coefficient, which depends on properties of materials and current temperature of annealing; temperature dependence of heat conduction coefficient in the most interesting temperature interval could be approximated by the following function \( \lambda(x, y, z, T) = \lambda_{ass}(x, y, z) \left[ 1 + \mu \left( T_d - T(x, y, z) \right)^\eta \right] \) (see, for example, [23]); \( p(x, y, z, t) \) is the volumetric density of heat power, gener-
ated in heterostructure during annealing: \( \alpha(x,y,z,T)=\lambda(x,y,z,T)/c(T) \) is the heat diffusivity. First of all we determine spatio-temporal distribution of temperature. To calculate the distribution of temperature we used recently introduced approach [24-26]. Framework the approach we transform approximation of heat diffusivity to the following form: \( \alpha_{\text{ass}}(x,y,z)=\lambda_{\text{ass}}(x,y,z)/c_{\text{ass}}=\alpha_{\text{ass}}[1+\varepsilon_T g_T(x,y,z)] \). Farther we determine solution of Eq.(8) as the following power series

\[
T(x,y,z,t) = \sum_{i=0}^{\infty} \varepsilon_i^{T} \sum_{j=0}^{\infty} \mu^T(x,y,z,t).
\]

Substitution of the series into Eq.(8) gives us possibility to obtain system of equations for the initial-order approximation of temperature \( T_{ij}(x,y,z,t) \) and corrections for them \( T_{ij}(x,y,z,t) \) (\( i \geq 1, j \geq 2 \)). The equations are presented in the Appendix. Substitution of the series (9) into boundary and initial conditions for temperature gives us possibility to obtain the same conditions for all functions \( T_{ij}(x,y,z,t) \) (\( i \geq 0, j \geq 0 \)). The conditions are presented in the Appendix. The equations for the functions \( T_{ij}(x,y,z,t) \) (\( i \geq 0, j \geq 0 \) with account boundary and initial conditions have been solved by using standard approaches [27,28] for the second-order approximation of the temperature \( T(x,y,z,t) \) on the parameters \( \varepsilon \) and \( \mu \). The solutions are presented in the Appendix. The second-order is usually enough good approximation to make qualitative analysis and to obtain some quantitative results (see, for example, [24-26]). Analytical results give us possibility to make more demonstrative analysis in comparison with numerical one. To calculate the obtained result with higher exactness and checking the obtain results by independent approaches we used numerical approaches.

To calculate spatio-temporal distributions of concentrations of point of radiation defects we used recently introduced approach [24-26] and transform approximations of diffusion coefficients in the following form: \( D_{ij}(x,y,z,T)=D_{ij}[1+\varepsilon_D g_D(x,y,z,T)] \), where \( D_{ij} \) are the average values of diffusion coefficients, \( 0 \leq \varepsilon_D \leq 1 \), \( |\varepsilon_D(x,y,z,T)| \leq 1 \), \( \rho = I^2V \). The same transformations have been used for approximations of parameters of recombination of point radiation defects and generation of their complexes: \( k_{iij}(x,y,z,T)=k_{ijij}[1+\varepsilon_{kij} g_{kij}(x,y,z,T)] \) and \( k_{ijij}(x,y,z,T)=k_{ijij}[1+\varepsilon_{kij} g_{kij}(x,y,z,T)] \), where \( k_{ijij} \) are the appropriate average values of these parameters, \( 0 \leq \varepsilon_{kij} \leq 1 \), \( 0 \leq \varepsilon_{kij} \leq 1 \), \( |g_{kij}(x,y,z,T)| \leq 1 \), \( |g_{kij}(x,y,z,T)| \leq 1 \). Let us introduce the following dimensionless variables: \( \bar{\nabla}(x,y,z,t)=I(x,y,z,t)/I^2, \chi=x/L, \eta=y/L, \phi = \varepsilon_D \). The introduction leads to the following transformation of equations (4) and conditions (5)

\[
\begin{align*}
\frac{\partial \bar{T}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} & = \frac{D_{ij}}{\sqrt{D_{ij}D_{gw}}} \frac{\partial}{\partial \chi} \left[ [1+\varepsilon_{ij} g_{ij}(\chi, \eta, \phi, T)] \frac{\partial \bar{T}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + \frac{D_{ij}}{\sqrt{D_{ij}D_{gw}}} \times \\
& \times \frac{\partial}{\partial \eta} \left[ [1+\varepsilon_{ij} g_{ij}(\chi, \eta, \phi, T)] \frac{\partial \bar{T}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \frac{D_{ij}}{\sqrt{D_{ij}D_{gw}}} \frac{\partial}{\partial \phi} \left[ [1+\varepsilon_{ij} g_{ij}(\chi, \eta, \phi, T)] \times \\
& \times \frac{\partial \bar{T}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] - \omega \left[ 1+\varepsilon_{ij} g_{ij}(\chi, \eta, \phi, T) \right] \bar{I}(\chi, \eta, \phi, \vartheta) \bar{V}(\chi, \eta, \phi, \vartheta) - \\
& - \Omega \left[ 1+\varepsilon_{ij} g_{ij}(\chi, \eta, \phi, T) \right] \bar{I}(\chi, \eta, \phi, \vartheta) \end{align*}
\]
standard approaches (see, for example, Fourier approach, [27, 28]). The equations are presented in the Appendix. The equations have been solved by determining approximations of diffusion coefficients into the following form:

$$\Phi(x, \eta, \phi, \vartheta) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \Omega_{ij} \rho \Phi(x, \eta, \phi, \vartheta).$$

We determine solutions of Eqs.(11) as the following power series (see [24-26])

$$\tilde{\rho}(x, \eta, \phi, \vartheta) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \omega_{ijk} \tilde{\rho}(x, \eta, \phi, \vartheta).$$

Substitution of the series (13) into Eqs. (11) and conditions (12) gives us possibility to obtain equations for initial-order approximations of concentrations of point defects $\tilde{\rho}_{ij}(x, \eta, \phi, \vartheta)$ and $\tilde{V}_{ij}(x, \eta, \phi, \vartheta)$ and corrections for them $\tilde{\rho}_{ijk}(x, \eta, \phi, \vartheta)$ and $\tilde{V}_{ijk}(x, \eta, \phi, \vartheta)$, $i \geq 1$, $j \geq 1$, $k \geq 1$. The equations and conditions for them are presented in the Appendix. The equations have been solved by standard approaches (see, for example, Fourier approach, [27, 28]). The equations are presented in the Appendix.

Further we determine spatio-temporal distributions of concentrations of complexes of radiation defects. First of all we transform approximations of diffusion coefficients into the following form:

$$D_\phi(x, y, z, T) = D_{0\phi} [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)],$$

where $D_{0\phi}$ are the average values of diffusion coefficients. In this situation Eqs.(6) will be transformed to the following form

$$\frac{\partial \Phi_i(x, y, z, t)}{\partial t} = D_{0\phi} \frac{\partial}{\partial x} \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial x} \right] + D_{0\phi} \frac{\partial}{\partial y} \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial y} \right] +$$

$$\times \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial z} \right] + D_{0\phi} \frac{\partial}{\partial x} \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial x} \right] +$$

$$\times \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial y} \right] + D_{0\phi} \frac{\partial}{\partial z} \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial z} \right] +$$

$$\times \left[ [1 + \varepsilon_{\phi\psi} g_\phi(x, y, z, T)] \frac{\partial \Phi_i(x, y, z, t)}{\partial z} \right] + k_{v\phi}(x, y, z, T) \Phi_i(x, y, z, t) - k_{v\phi}(x, y, z, T) \Phi_i(x, y, z, t).$$
Farther we determine solutions of the above equations as the following power series
\[
\Phi_\rho(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_i \Phi_{\rho i}(x, y, z, t).
\] (11)

Substitution of the series (14) into Eqs. (6) and appropriate boundary and initial conditions gives us possibility to obtain equations for initial-order approximations of concentrations of complexes of radiation defects \( \Phi_{\rho 0}(x, y, z, t) \) and corrections for them \( \Phi_{\rho i}(x, y, z, t) \) \((i \geq 1)\) and appropriate conditions for all functions \( \Phi_{\rho i}(x, y, z, t) \) \((i \geq 0)\). The equations and conditions are presented in the Appendix. The obtained equations have been solved by standard approaches (see, for example, [27,28]) with account boundary and initial conditions. The solutions are presented in the Appendix.

We calculate spatio-temporal distribution of dopant concentration by using the same approach, which have been used for calculation spatio-temporal distribution of concentrations of radiation defects. In this situation we transform approximation of dopant diffusion coefficient to the following form:
\[
D_L(x,y,z,T) = D_{0L}[1+\xi L(x,y,z,T)],
\]
where \( D_{0L} \) is the average value of dopant diffusion coefficient, \( 0 \leq \xi L < 1 \), \( |\xi L(x,y,z,T)| \leq 1 \). Farther we determine solution of Eq. (1) in the following form
\[
C(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_i \sum_{j=0}^{\infty} \xi_j C_{ij}(x, y, z, t).
\]

Substitution of the series into Eq. (1) and conditions (2) gives us possibility to obtain equation for initial-order approximation of the concentration of dopant \( C_{00}(x, y, z, t) \) and corrections for them \( C_{ij}(x, y, z, t) \) \((i \geq 1, j \geq 1)\) and boundary and initial conditions for them. The equations and conditions are presented in the Appendix. The solutions have been calculated by standard approaches (see, for example, [27,28]). The solutions are presented in the Appendix.

Analysis of spatio-temporal distributions of concentrations of dopant and radiation defects have been done analytically by using the second-order approximations on all parameters, which are used in appropriate series. The approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. Results of analytical calculations have been checked with comparison with numerical one.

3. Discussion

In this section we analyzed redistribution of dopant and radiation defects by using relations, calculated in the previous section. Typical distributions of concentrations of dopant near interface between materials of heterostructure are presented on Figs. 2 and 3 for diffusion and ion types of doping, respectively. The distributions have been calculated for the case, when value of dopant diffusion coefficient in doped area is larger, than value of dopant diffusion coefficient in nearest areas. The figures show, that presents of interface between materials gives us possibility to increase sharpness of \( p-n \)-junctions, which included into the considered heterobipolar transistor. At the same time homogeneity of distribution of concentration of dopant increases. Increasing of sharpness of \( p-n \)-junctions gives us possibility to decrease their switching time. Increasing of homogeneity of distribution of concentration of dopant gives us possibility to decrease value of local overheats during functioning of the \( p-n \)-junctions or to decrease dimensions of \( p-n \)-junctions with fixed maximal value of the overheats. To accelerate transport of charge carriers it is attracted an interest inhomogenous distribution of dopant in base. In this case it is electrical field has been generated in the base. This electrical field gives us possibility to accelerate transport of charge carriers in base of transistors. To manufacture in homogenous distribution of dopant in base it is practicably to dope required area (section) of the first (nearest to the substrate) epitaxial layer. After that it is practicably to anneal dopant and/or radiation defects. Farther they are attracted an
interest the following steps: (i) manufacturing of the second epitaxial layer with section, manufactured by using another materials; (ii) doping the section of the second epitaxial layer by diffusion or ion implantation; (iii) manufacturing of the third epitaxial layer with section, manufactured by using another materials; (iv) doping the section of the third epitaxial layer by diffusion or ion implantation. After that we consider microwave annealing of dopant and/or radiation defects. Advantage of the approach of annealing is formation of inhomogenous distribution of temperature. In this situation it is practicably to choose parameters of annealing so, that thickness of scin-layer became larger, than thickness of the third (external) epitaxial layer and smaller, than total of thickness of the third and the second epitaxial layers. In this case dopant diffusion in nearest to the substrate side became slower, than in farther side. This is a reason to inhomogeneity of distribution of concentration of dopant in depth of hetero structure. After finishing of manufacturing of bipolar transistor the section of the average epitaxial layer with inhomogenous distribution of concentration of dopant assumes function of base.

Fig. 2. Distributions of concentrations of infused dopant in hetero structure from Fig. 1 in direction, which is perpendicular to interface between layers of heterostructure. Increasing of number of curves corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure. The curves have been calculated under condition, when dopant diffusion coefficient in doped layer is larger, than in nearest layer.
Fig. 3. Spatial distributions of implanted dopant concentration after annealing with continuous $\Theta = 0.0048(L_x^2+L_y^2+L_z^2)/D_0$ (curves 1 and 3) and $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$ (curves 2 and 4). Curves 1 and 2 are calculated distributions of dopant concentration in homogenous structure. Curves 3 and 4 are calculated distributions of dopant concentration in hetero structure under condition, when dopant diffusion coefficient in doped layer is larger, than in nearest layer. Using of the considered approach to manufacture of transistors leads to necessity of optimization of annealing time. To optimize the annealing time we used recently introduced criterion [24,26,29-33]. Framework the approach we approximate real distributions of concentration of dopant by step-wise function. Farther we determine the required optimal values of annealing time by minimization of the following mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_{0}^{L_x} \int_{0}^{L_y} \int_{0}^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)] d z d y d x, \quad (15)$$

where $\psi(x)$ is the approximation function, $\Theta$ is the required value of annealing time.

Fig.4. Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation $a/L$ and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of hetero structure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter $\varepsilon$ for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter $\xi$ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter $\gamma$ for $a/L=1/2$ and $\varepsilon = \xi = 0$.
Fig.5. Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation \(a/L\) and \(\xi = \gamma = 0\) for equal to each other values of dopant diffusion coefficient in all parts of hetero structure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter \(\varepsilon\) for \(a/L=1/2\) and \(\xi = \gamma = 0\). Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter \(\xi\) for \(a/L=1/2\) and \(\varepsilon = \gamma = 0\). Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter \(\gamma\) for \(a/L=1/2\) and \(\varepsilon = \xi = 0\).

Dependences of optimal values of annealing time are presented in Fig. 4 for diffusion type of doping. Using ion implantation leads to necessity of annealing of radiation defects. In the ideal case after finishing of annealing of radiation defects dopant achieves interface between materials of hetero structure. If the dopant did not achieves the interface during the annealing, it is practically to use additional annealing of dopant. Dependences of optimal values of additional annealing time are presented in Fig. 5. Optimal value of time of additional annealing of implanted dopant is smaller, than in optimal value of infused dopant. Reason of this difference is necessity of annealing of radiation defects.

4. CONCLUSIONS

In this paper we introduce an approach to manufacture a heterobipolar transistor with inhomogeneous doping of base. At the same time the introduced approach to manufacture of bipolar transistors gives us possibility to increase their compactness and to increase sharpness of \(p-n\)-junctions, which included into the transistor. The approach based on manufacturing of a heterostructure with special construction, doping of special areas of the hetero structure and optimization of annealing of dopant and/or radiation defects.

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APPENDIX

Equations for the functions $T_{ij}(x,y,z,t)$ ($i\geq 0, j\geq 0$) have been obtained by substitution the power series (10) in the equation (8) and equating terms with equal powers of parameters $\varepsilon_j$ and $\mu$. The equations could be written as

$$\frac{\partial T_{00}(x,y,z,t)}{\partial t} = \alpha_{000} \left[ \frac{\partial^3 T_{00}(x,y,z,t)}{\partial x^3} + \frac{\partial^2 T_{00}(x,y,z,t)}{\partial x \partial y} + \frac{\partial^2 T_{00}(x,y,z,t)}{\partial x \partial z} \right] + \frac{\partial T_{00}(x,y,z,t)}{v_{00}},$$

$$\frac{\partial T_{ij}(x,y,z,t)}{\partial t} = \alpha_{ij} \left[ \frac{\partial^3 T_{ij}(x,y,z,t)}{\partial x^3} + \frac{\partial^2 T_{ij}(x,y,z,t)}{\partial x \partial y} + \frac{\partial^2 T_{ij}(x,y,z,t)}{\partial x \partial z} \right] + \alpha_{ij} T_d \times$$

$$\times \left\{ g_r(x,y,z,T) \frac{\partial^2 T_{ij}(x,y,z,t)}{\partial x^2} + g_r(x,y,z,T) \frac{\partial^2 T_{ij}(x,y,z,t)}{\partial y^2} + \frac{\partial}{\partial z} \left[ g_r(x,y,z,T) \frac{\partial T_{ij}(x,y,z,t)}{\partial z} \right] \right\}, \quad i \geq 1$$

$$\frac{\partial T_{01}(x,y,z,t)}{\partial t} = \alpha_{001} \left[ \frac{\partial^3 T_{01}(x,y,z,t)}{\partial x^3} + \frac{\partial^2 T_{01}(x,y,z,t)}{\partial x \partial y} + \frac{\partial^2 T_{01}(x,y,z,t)}{\partial x \partial z} \right] + \alpha_{001} T_d \times$$

$$\times \left[ \frac{\partial^2 T_{01}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 T_{01}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 T_{01}(x,y,z,t)}{\partial z^2} \right] + \frac{\partial}{\partial z} \left[ T_{01}(x,y,z,t) \frac{\partial T_{01}(x,y,z,t)}{\partial z} \right]^2 +$$

$$+ \left\{ \left[ \frac{\partial T_{01}(x,y,z,t)}{\partial x} \right]^2 + \left[ \frac{\partial T_{01}(x,y,z,t)}{\partial y} \right]^2 \right\}$$

$$\frac{\partial T_{02}(x,y,z,t)}{\partial t} = \alpha_{002} \left[ \frac{\partial^3 T_{02}(x,y,z,t)}{\partial x^3} + \frac{\partial^2 T_{02}(x,y,z,t)}{\partial x \partial y} + \frac{\partial^2 T_{02}(x,y,z,t)}{\partial x \partial z} \right] + \alpha_{002} T_d \times$$

$$\times \left[ \frac{\partial^2 T_{02}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 T_{02}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 T_{02}(x,y,z,t)}{\partial z^2} \right] + \frac{\partial}{\partial y} \left[ T_{02}(x,y,z,t) \frac{\partial T_{02}(x,y,z,t)}{\partial y} \right] +$$

$$+ \frac{\partial}{\partial z} \left[ T_{02}(x,y,z,t) \frac{\partial T_{02}(x,y,z,t)}{\partial z} \right]$$

$$\frac{\partial T_{11}(x,y,z,t)}{\partial t} = \alpha_{11} \frac{\partial T_{11}(x,y,z,t)}{\partial x} + \alpha_{11} T_{01}(x,y,z,t) \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial x} + \frac{\partial^2 T_{01}(x,y,z,t)}{\partial y^2} \right] \times$$

$$\times g_r(x,y,z,T) + \frac{\partial}{\partial z} \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial z} \right] + \alpha_{11} \left\{ \left. \frac{\partial}{\partial x} \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial x} \right] \right| +$$

$$+ \frac{\partial}{\partial y} \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial y} \right] + \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial z} \right] \right\} +$$

$$+ \frac{\alpha_{11}}{T_{01}(x,y,z,t)} \left. \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial x} \right] \right| + \frac{\alpha_{11}}{T_{01}(x,y,z,t)} \left. \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial y} \right] \right| +$$

$$+ \frac{\alpha_{11}}{T_{01}(x,y,z,t)} \left. \left[ g_r(x,y,z,T) \frac{\partial T_{01}(x,y,z,t)}{\partial z} \right] \right| \times$$

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\[ T_{00}(x, y, z, t) + \alpha_{\text{max}} T_0^\phi \left( \frac{\partial^2 T_{10}}{\partial z^2} \right) \] \\
\[ + \frac{\partial^2 T_{10}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 T_{10}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 T_{10}(x, y, z, t)}{\partial z^2} \] \\
\[ + \left\{ g_{\tau}(x, y, z, T) \frac{\partial^2 T_{10}}{\partial x^2} \right\} \] \\
\[ + \frac{\partial}{\partial z} \left\{ g_{\tau}(x, y, z, T) \frac{\partial T_{00}}{\partial z} \right\} + g_{\tau}(x, y, z, T) \times \] \\
\[ \frac{\alpha_{\text{max}}}{T_{00}(x, y, z, t)} \frac{\partial^2 T_{10}}{\partial x^2} \] \\
\[ + \frac{\partial T_{10}}{\partial x} \frac{\partial T_{00}}{\partial x} + \frac{\partial T_{10}}{\partial y} \frac{\partial T_{00}}{\partial y} \times \] \\
\[ \frac{\partial T_{00}}{\partial x} \] \\
\[ \left\{ \left( \frac{\partial T_{10}}{\partial x} \right)^2 + \left( \frac{\partial T_{10}}{\partial y} \right)^2 + \left( \frac{\partial T_{10}}{\partial z} \right)^2 \right\}. \]

Conditions for the functions \( T_{ij}(x, y, z, t) \) (\( i \geq 0, j \geq 0 \)) have been obtained by the same procedure as appropriate equations and could be written as

\[ \frac{\partial T_{ij}}{\partial x} \bigg|_{x=0} = 0, \quad \frac{\partial T_{ij}}{\partial x} \bigg|_{x=L_x} = 0, \quad \frac{\partial T_{ij}}{\partial y} \bigg|_{y=0} = 0, \quad \frac{\partial T_{ij}}{\partial y} \bigg|_{y=L_y} = 0, \]

Solutions of the equations for the functions \( T_{ij}(x, y, z, t) \) (\( i \geq 0, j \geq 0 \)) with account boundary and initial conditions have been obtained by standard Fourier approach. By using the approach one can obtain the functions \( T_{ij}(x, y, z, t) \) in the following form

\[ T_{00}(x, y, z, t) = \frac{1}{L_x L_y L_z} \sum_{n=0}^{L_x} c_n(x) c_n(y) c_n(z) \sum_{n=1}^{L_z} c_n(u) \int_0^{L_x} f_i(u, v, w) d w d v d u + \frac{2}{L_x L_y L_z} \sum_{n=1}^{L_z} c_n(x) c_n(y) c_n(z) e_{n \alpha}(\tau) \sum_{n=1}^{L_z} c_n(u) \int_0^{L_z} f_i(u, v, w) d w d v d u \]

\[ \times \frac{1}{v_{\alpha}} \int_0^{L_x} p(u, v, w, \tau) d w d v d u d \tau + \frac{2}{L_x L_y L_z} \int_0^{L_x} \sum_{n=1}^{L_z} c_n(y) c_n(z) e_{n \alpha}(\tau) \int_0^{L_z} c_n(u) c_n(v) \frac{d(u, v, w, \tau)}{v_{\alpha}} d w d v d u d \tau, \]

where \( c_n(\chi) = \cos(\pi n \chi L), \quad e_{n \alpha}(\tau) = \exp \left\{ -\pi^2 n^2 \alpha_{\text{max}} \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \right\}; \]

\[ \frac{\partial T_{00}}{\partial u} \int_0^{L_x} f_i(u, v, w) d w d v d u d \tau + \frac{2}{L_x L_y L_z} \sum_{n=1}^{L_z} c_n(x) c_n(y) c_n(z) e_{n \alpha}(\tau) \int_0^{L_z} c_n(u) c_n(v) \frac{d(u, v, w, \tau)}{v_{\alpha}} d w d v d u d \tau \]

\[ \times \frac{\partial^2 T_{00}}{\partial u \partial v} \int_0^{L_x} f_i(u, v, w) d w d v d u d \tau + \frac{2}{L_x L_y L_z} \sum_{n=1}^{L_z} c_n(x) c_n(y) c_n(z) e_{n \alpha}(\tau) \int_0^{L_z} c_n(u) c_n(v) \frac{d(u, v, w, \tau)}{v_{\alpha}} d w d v d u d \tau \times \]
\[ \times \int_{0}^{l_z} \left[ c_n(w) g_y(u,v,w,T) \frac{\partial T_{i-10}(u,v,w,\tau)}{\partial v} \right] dwdvdud\tau + \sum_{n=1}^{\infty} \frac{\pi \alpha_{\text{max}}}{L_x L_y L_z} c_n(x)c_n(y)c_n(z) \times \]

\[ \times e_{\text{st}}(t) \left[ e_{\text{st}}(-\tau) \frac{l_z}{l_z} c_n(u) \frac{l_z}{l_z} c_n(v) \right] s_n(w) g_y(u,v,w,T) \frac{\partial T_{i-10}(u,v,w,\tau)}{\partial w} dwdvdud\tau, \quad i \geq 1, \]

where \( s_n(\chi) = \sin(\pi \chi/L); \)
\[ T_{01}(x,y,z,t) = \frac{2\pi T_0^\phi}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times \frac{\partial^2 T_{01}(u,v,w,\tau)}{\partial u^2} + \frac{2\pi T_0^\phi}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times n \frac{l_z}{l_z} c_n(w) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} s_n(w) \frac{\partial^3 T_{01}(u,v,w,\tau)}{\partial u^3} \]

\[ \times \frac{2\pi T_0^\phi}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \]

\[ \left( \frac{\partial \theta_{01}(u,v,w,\tau)}{\partial w} \right)^2 \int_{0}^{l_z} \left( \frac{\partial \theta_{01}(u,v,w,\tau)}{\partial w} \right)^2 dwdvdud\tau, \]

\[ T_{02}(x,y,z,t) = \frac{2\pi T_0^\phi}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times \frac{\partial^3 T_{02}(u,v,w,\tau)}{\partial u^3} \frac{T_{02}(u,v,w,\tau)}{T_{02}(u,v,w,\tau)} + \sum_{n=1}^{\infty} \frac{\pi \alpha_{\text{max}}}{L_x L_y L_z} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times \frac{\partial^2 T_{02}(u,v,w,\tau)}{\partial v^2} + \sum_{n=1}^{\infty} \frac{\pi \alpha_{\text{max}}}{L_x L_y L_z} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times \frac{\partial T_{02}(u,v,w,\tau)}{\partial w} + \sum_{n=1}^{\infty} \frac{\pi \alpha_{\text{max}}}{L_x L_y L_z} c_n(x)c_n(y)c_n(z) e_{\text{st}}(t) \int_{0}^{l_z} s_n(u) \frac{l_z}{l_z} c_n(v) \frac{l_z}{l_z} c_n(w) \times \]

\[ \times \frac{\partial T_{02}(u,v,w,\tau)}{\partial w} \int_{0}^{l_z} \left( \frac{\partial \theta_{02}(u,v,w,\tau)}{\partial w} \right)^2 dwdvdud\tau, \]

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\[
\begin{align*}
\times & \frac{\partial T_{00y}(u,v,w) \partial T_{00x}(u,v,w,\tau) \partial T_{00z}(u,v,w,\tau)}{\partial \tau} \frac{d w d v d u d \tau}{T_{00y}(u,v,w,\tau)} - \frac{2 \pi \alpha_{\text{max}}}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x) c_n(y) c_n(z) e_{\text{ct}}(t) \times \\
\times & T_{00y}(u,v,w,\tau) T_{00x}(u,v,w,\tau) w_{\tau}(u,v,w,\tau) \frac{d w d v d u d \tau}{T_{00y}(u,v,w,\tau)} \\
T_{01}(x,y,z \tau) = & \frac{2 \alpha_{\text{max}}}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x) c_n(y) c_n(z) e_{\text{ct}}(t) e_{\text{et}}(\tau) \frac{d w d v d u d \tau}{T_{00y}(u,v,w,\tau)} \\
\times & \frac{\partial^2 T_{00y}(u,v,w,\tau)}{\partial u^2} + \frac{\partial^2 T_{00x}(u,v,w,\tau)}{\partial v^2} + \frac{\partial^2 T_{00z}(u,v,w,\tau)}{\partial w^2} + 2 \frac{\alpha_{\text{max}} T_{00}^y}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x) c_n(y) c_n(z) \times \\
\times & \frac{\partial^2 T_{00y}(u,v,w,\tau)}{\partial u^2} + \frac{\partial^2 T_{00x}(u,v,w,\tau)}{\partial v^2} + \frac{\partial^2 T_{00z}(u,v,w,\tau)}{\partial w^2} + 2 \frac{\alpha_{\text{max}} T_{00}^y}{L_x L_y L_z} \sum_{n=1}^{\infty} c_n(x) c_n(y) c_n(z) \times \\
\times & \frac{\partial^2 T_{00y}(u,v,w,\tau)}{\partial u^2} + \frac{\partial^2 T_{00x}(u,v,w,\tau)}{\partial v^2} + \frac{\partial^2 T_{00z}(u,v,w,\tau)}{\partial w^2}
\end{align*}
\]
Equations for the functions $\tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)$ and $\tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)$, $i \geq 0, j \geq 0, k \geq 0$ and conditions for them have been obtain by the same procedure as for the functions $T_{ij}(x, y, z, t)$

$$\frac{\partial \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{I}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{I}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{I}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{I}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{V}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{V}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{V}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{V}_{0i}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{I}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{V}_{i0}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{I}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$

$$\frac{\partial \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = \frac{D_{x}}{D_{x}x} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^{2}} + \frac{D_{y}}{D_{y}y} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^{2}} + \frac{D_{z}}{D_{z}z} \frac{\partial^{2} \tilde{V}_{00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta^{2}}$$
\[
\frac{\partial T_{000}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] - \left[ 1 + \epsilon_{i,j} \frac{g_{ij}(\chi, \eta, \phi, \theta)}{\partial \chi^2} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial T_{000}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 T_{000}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] - \left[ 1 + \epsilon_{i,j} \frac{g_{ij}(\chi, \eta, \phi, \theta)}{\partial \chi^2} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] + \frac{\partial}{\partial \chi} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \eta} \right] + \frac{\partial}{\partial \phi} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \phi} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] - \left[ 1 + \epsilon_{i,j} \frac{g_{ij}(\chi, \eta, \phi, \theta)}{\partial \chi^2} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] - \left[ 1 + \epsilon_{i,j} \frac{g_{ij}(\chi, \eta, \phi, \theta)}{\partial \chi^2} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] + \frac{\partial}{\partial \chi} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \eta} \right] + \frac{\partial}{\partial \phi} \left[ g_{ij}(\chi, \eta, \phi, \theta) \frac{\partial \tilde{T}_{10}(\chi, \eta, \phi, \theta)}{\partial \phi} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]

\[
\frac{\partial \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \theta} = \sqrt{\frac{D_{uv}}{D_{ov}}} \left[ \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \chi^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \eta^2} + \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \theta)}{\partial \phi^2} \right] - \left[ 1 + \epsilon_{i,j} \frac{g_{ij}(\chi, \eta, \phi, \theta)}{\partial \chi^2} \right] \tilde{T}_{000}(\chi, \eta, \phi, \theta)
\]
Solutions of the above equations have been obtained by standard Fourier approach and could be written as

\[ \tilde{\rho}_{000}(\chi, \eta, \phi, \theta) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} F_{np} c(\chi)c(\eta)c(\phi)e_{np}(\theta), \]

where \[ F_{np} = \frac{1}{\rho^2} \int \cos(nu) \cos(nv) \cos(nw) f_{np}(u,v,w) \, dv \, dw \, du, \]
\[ c(\chi) = \cos(\pi \chi), \]
\[ c_{m}(\theta) = \exp(-\pi^2 n^2 \rho^2 / D_{0w} / D_{0\eta}), \]
\[ e_{np}(\theta) = \exp(-\pi^2 n^2 \rho^2 / D_{0w} / D_{0\eta}); \]
\[ \tilde{I}_{000}(\chi, \eta, \phi, \theta) = -2\pi \sum_{n=1}^{\infty} n c_{n}(\chi)c(\eta)c(\phi)e_{n\phi}(\theta) \int_{0}^{\pi} \int_{0}^{\pi} \int_{0}^{1} c_{n}(w) g_{n}(u,v,w,T) \times \]
\[ \times c_{n}(w) \frac{\partial \tilde{I}_{000}(u,v,w,T)}{\partial u} \, dv \, dw \, du \, d\tau - 2 \frac{D_{0w}}{D_{0\eta}} \sum_{n=1}^{\infty} n c_{n}(\chi)c(\eta)c(\phi)e_{n\phi}(\theta) \int_{0}^{\pi} \int_{0}^{\pi} \int_{0}^{1} c_{n}(w) g_{n}(u,v,w,T) \times \]
\[ \times \pi \left[ s_n(v) \right]_0^1 c_n(w) g_i(u,v,w,T) \frac{\partial \tilde{T}_{i=00}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{0v}}{D_{0w}}} \sum_{n=1}^{\infty} n c_n(\chi)c(\eta)c(\phi) \times \\
\times e_{sd}(\phi) e_{sd}(\phi) c_n(u) c_n(v) \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau \tau
\]
\[ \times c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) - 2 \sum_{n=1}^{\infty} c_\ast (\chi)e_{\ast \ast} (\theta) c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) [1 + \varepsilon_{\ast \ast \ast}] \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \int \mathcal{I}_{100} (u, v, w, \tau) \mathcal{I}_{100} (u, v, w, \tau) + \mathcal{I}_{100} (u, v, w, \tau) \mathcal{I}_{100} (u, v, w, \tau) \int d w d v d u d \tau \]
\[ \mathcal{V}_{100} (\chi, \eta, \phi, \theta) = -2\pi \sqrt{\frac{D_{uv}}{D_{uv}}} \int c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (\theta) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \frac{\partial \mathcal{V}_{100} (u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{uv}}{D_{uv}}} \int c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (\theta) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \frac{\partial \mathcal{V}_{100} (u, v, w, \tau)}{\partial v} d w d v d u d \tau = -2\pi \sqrt{\frac{D_{uv}}{D_{uv}}} \int \sum_{n=1}^{\infty} c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (\theta) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \frac{\partial \mathcal{I}_{100} (u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{uv}}{D_{uv}}} \int \sum_{n=1}^{\infty} c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (\theta) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \frac{\partial \mathcal{I}_{100} (u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi \sqrt{\frac{D_{uv}}{D_{uv}}} \int \sum_{n=1}^{\infty} c_\ast (\chi)c_\ast (\eta)c_\ast (\phi) e_{\ast \ast} (\theta) e_{\ast \ast} (-\tau) \int c_\ast (a)\int c_\ast (v)\int c_\ast (w) \times \]
\[ \times g_{\ast \ast \ast \ast} (u, v, w, T) \frac{\partial \mathcal{V}_{100} (u, v, w, \tau)}{\partial w} d w d v d u d \tau \]
\[
\times c_n(\chi)c_n(\eta)c_n(\phi) - 2 \sum_{n=0}^{\infty} c_n(\chi)c_n(\eta)c_n(\phi) e_n(\varepsilon_0) e_n(-\tau) \int c_n(u) c_n(v) c_n(w) \left[ 1 + \varepsilon_{i,y} \times \right.
\times g_{f,x}(u,v,w) \tilde{T}_{\alpha}(u,v,w,\tau) \tilde{V}_{\alpha}(u,v,w,\tau) \right] 
\times \tilde{T}_{\alpha}(u,v,w,\tau) \tilde{V}_{\alpha}(u,v,w,\tau) \right] dwdvdud\tau ;
\]

\[
\tilde{I}_{\alpha}(\chi,\eta,\phi,\varepsilon) = -2 \sum_{n=0}^{\infty} c_n(\chi)c_n(\eta)c_n(\phi) e_n(\varepsilon_0) e_n(-\tau) \int c_n(u) c_n(v) c_n(w) \times \left[
\left[ 1 + \varepsilon_{i,y} g_{f,x}(u,v,w,\tau) \tilde{T}_{\alpha}(u,v,w,\tau) \right] \times \tilde{T}_{\alpha}(u,v,w,\tau) \tilde{V}_{\alpha}(u,v,w,\tau) \right] dwdvdud\tau ;
\]

Equations for initial-order approximations of distributions of concentrations of simplest complexes of radiation defects \( \Phi_{\alpha}(x,y,z,t) \) and corrections for them \( \Phi_{\beta}(x,y,z,t) \), \( i \geq 1 \) and boundary and initial conditions for them have been obtained as the functions \( T_{\alpha}(x,y,z,t) \) and takes the form

\[
\frac{\partial \Phi_{\alpha}(x,y,z,t)}{\partial t} = D_{\Phi_{\alpha}} \left[ \frac{\partial^2 \Phi_{\alpha}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \Phi_{\alpha}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \Phi_{\alpha}(x,y,z,t)}{\partial z^2} \right] +
+k_{i,j}(x,y,z,T) I^2(x,y,z,t) - k_j(x,y,z,T) I(x,y,z,t) ;
\]

\[
\frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial t} = D_{\Phi_{\beta}} \left[ \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial z^2} \right] +
+k_{i,j}(x,y,z,T) V^2(x,y,z,t) - k_j(x,y,z,T) V(x,y,z,t) ;
\]

where \( i \geq 1 \),

\[
\frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial t} = D_{\Phi_{\beta}} \left[ \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial z^2} \right] +
+k_{i,j}(x,y,z,T) \left\{ \phi g_{\Phi}(x,y,z,T) \frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial x} \right\} , i \geq 1 ;
\]

\[
\frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial t} = D_{\Phi_{\beta}} \left[ \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial z^2} \right] +
+k_{i,j}(x,y,z,T) \left\{ \phi g_{\Phi}(x,y,z,T) \frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial y} \right\} , i \geq 1 ;
\]

\[
\frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial t} = D_{\Phi_{\beta}} \left[ \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \Phi_{\beta}(x,y,z,t)}{\partial z^2} \right] +
+k_{i,j}(x,y,z,T) \left\{ \phi g_{\Phi}(x,y,z,T) \frac{\partial \Phi_{\beta}(x,y,z,t)}{\partial z} \right\} , i \geq 1 ;
\]
\[ \frac{\partial \Phi_{\rho_0}(x, y, z, t)}{\partial x} \bigg|_{x=0} = 0, \quad \frac{\partial \Phi_{\rho_0}(x, y, z, t)}{\partial y} \bigg|_{y=0} = 0, \quad \frac{\partial \Phi_{\rho_0}(x, y, z, t)}{\partial z} \bigg|_{z=0} = 0, \quad \frac{\partial \Phi_{\rho_0}(x, y, z, t)}{\partial t} \bigg|_{t=0} = 0, \quad \forall \rho > 0; \]

\[ \Phi_{\rho_0}(x, y, z, 0) = f_{\rho_0}(x, y, z), \quad \Phi_{\rho_0}(x, y, z, 0) = 0, \quad \forall \rho > 0. \]

Solutions of the above equations could be written as

\[ \Phi_{\rho}(x, y, z, t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{i=1}^{n} E_{\rho_0} c_i(x) c_i(y) c_i(z) e_{\rho_0}^2(t) + \frac{2}{L_x L_y L_z} \sum_{i=1}^{n} n c_i(x) c_i(y) c_i(z) \times \]

\[ \times e_{\rho_0}^2(t) \sum_{i=1}^{l_{\rho_0}} (u(t) + l_{\rho_0}^2) c_i(v) c_i(w) [k_{i,j}(u, v, w, T) I^2(u, v, w, T) - k_{i,j}(u, v, w, T) \times \]

\[ \times I(u, v, w, T)] dw dv du \]

where \[ F_{\rho_0} = \sum_{i=1}^{l_{\rho_0}} c_i(v) c_i(w)f_{\rho_0}(u, v, w) dw dv du, \quad e_{\rho_0}^2(t) = \exp \left[ -\pi^2 n^2 D_{\rho_0} t \right], \]

\[ c_i(x) = \cos(\pi n x / L_x); \]

\[ \Phi_{\rho}(x, y, z, t) = -\frac{2}{L_x L_y L_z} \sum_{i=1}^{n} n c_i(x) c_i(y) c_i(z) e_{\rho_0}^2(t) \sum_{i=1}^{l_{\rho_0}} (u(t) + l_{\rho_0}^2) c_i(v) c_i(w) \times \]

\[ \times g_{\rho_0}(u, v, w, T) \frac{\partial \Phi_{i,j}(u, v, w, T)}{\partial u} dw dv du \]

\[ \times \frac{2}{L_x L_y L_z} \sum_{i=1}^{n} n c_i(x) c_i(y) c_i(z) e_{\rho_0}^2(t) \times \]

\[ \times \frac{1}{L_x L_y L_z} \sum_{i=1}^{l_{\rho_0}} c_i(v) c_i(w) \frac{\partial \Phi_{i,j}(u, v, w, T)}{\partial v} dw dv du \]

\[ \times \frac{2}{L_x L_y L_z} \sum_{i=1}^{n} n c_i(x) c_i(y) c_i(z) e_{\rho_0}^2(t) \times \]

\[ \times \frac{1}{L_x L_y L_z} \sum_{i=1}^{l_{\rho_0}} c_i(v) c_i(w) \frac{\partial \Phi_{i,j}(u, v, w, T)}{\partial w} dw dv du \]

where \[ s_{\rho}(x) = \sin(\pi n x / L_x). \]

Equations for initial-order approximation of dopant concentration \[ C_{\rho_0}(x, y, z, t) \], corrections for them \[ C_{\rho_0}(x, y, z, t) \] (\( i \geq 1, j \geq 1 \)) and boundary and initial conditions take the form

\[ \frac{\partial C_{\rho_0}(x, y, z, t)}{\partial t} = D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial x^2} + D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial y^2} + D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial z^2}; \]

\[ \frac{\partial C_{\rho_0}(x, y, z, t)}{\partial t} = D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial x^2} + D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial y^2} + D_{\rho_0} \frac{\partial^2 C_{\rho_0}(x, y, z, t)}{\partial z^2}; \]

\[ + D_{\rho_0} \frac{\partial}{\partial x} \left[ g_L(x, y, z, T) \frac{\partial C_{\rho_0}(x, y, z, t)}{\partial x} \right] + D_{\rho_0} \frac{\partial}{\partial y} \left[ g_L(x, y, z, T) \frac{\partial C_{\rho_0}(x, y, z, t)}{\partial y} \right] + \]

\[ + D_{\rho_0} \frac{\partial}{\partial z} \left[ g_L(x, y, z, T) \frac{\partial C_{\rho_0}(x, y, z, t)}{\partial z} \right] \]
\[
+ D_{\alpha l} \frac{\partial}{\partial z} \left[ g_i(x, y, z, T) \frac{\partial C_{i, \alpha l}(x, y, z, t)}{\partial z} \right], \quad i \geq 1;
\]

\[
\frac{\partial C_{\alpha l}(x, y, z, t)}{\partial t} = D_{\alpha l} \frac{\partial^2 C_{\alpha l}(x, y, z, t)}{\partial x^2} + D_{\alpha l} \frac{\partial^2 C_{\alpha l}(x, y, z, t)}{\partial y^2} + D_{\alpha l} \frac{\partial^2 C_{\alpha l}(x, y, z, t)}{\partial z^2} +
\]

\[
+ D_{\alpha l} \frac{\partial}{\partial x} \left[ C_{\alpha l}^\alpha(x, y, z, t) \frac{\partial C_{\alpha l}(x, y, z, t)}{\partial x} \right] + D_{\alpha l} \frac{\partial}{\partial y} \left[ C_{\alpha l}^\alpha(x, y, z, t) \frac{\partial C_{\alpha l}(x, y, z, t)}{\partial y} \right] +
\]

\[
+ D_{\alpha l} \frac{\partial}{\partial z} \left[ C_{\alpha l}^\alpha(x, y, z, t) \frac{\partial C_{\alpha l}(x, y, z, t)}{\partial z} \right];
\]

\[
\frac{\partial C_{1l}(x, y, z, t)}{\partial t} = D_{\alpha l} \frac{\partial^2 C_{1l}(x, y, z, t)}{\partial x^2} + D_{\alpha l} \frac{\partial^2 C_{1l}(x, y, z, t)}{\partial y^2} + D_{\alpha l} \frac{\partial^2 C_{1l}(x, y, z, t)}{\partial z^2} +
\]

\[
+ \frac{\partial}{\partial x} \left[ C_{1l}(x, y, z, t) \frac{C_{1l}^\alpha(x, y, z, t) \frac{\partial C_{1l}(x, y, z, t)}{\partial x}}{P^\alpha(x, y, z, T)} \right] + \frac{\partial}{\partial y} \left[ C_{1l}(x, y, z, t) \frac{C_{1l}^\alpha(x, y, z, t) \frac{\partial C_{1l}(x, y, z, t)}{\partial y}}{P^\alpha(x, y, z, T)} \right] +
\]

\[
+ \frac{\partial}{\partial z} \left[ C_{1l}(x, y, z, t) \frac{C_{1l}^\alpha(x, y, z, t) \frac{\partial C_{1l}(x, y, z, t)}{\partial z}}{P^\alpha(x, y, z, T)} \right];
\]

\[
\frac{\partial C_i(x, y, z, t)}{\partial x} \bigg|_{t=0} = 0, \quad \frac{\partial C_y(x, y, z, t)}{\partial x} \bigg|_{t=0} = 0, \quad \frac{\partial C_y(x, y, z, t)}{\partial y} \bigg|_{t=0} = 0.
\]
\[
\frac{\partial C_0(x,y,z,t)}{\partial y} \bigg|_{y=L_y} = 0, \quad \frac{\partial C_0(x,y,z,t)}{\partial z} \bigg|_{z=L_z} = 0, \quad \frac{\partial C_0(x,y,z,t)}{\partial z} \bigg|_{z=0} = 0, \quad i \geq 0, \ j \geq 0; \\
C_{00}(x,y,z,0) = f_c(x,y), \ C_{0j}(x,y,z,0) = 0, \ i \geq 1, \ j \geq 1.
\]

Solutions of the above equations with account boundary and initial conditions could be written as

\[
C_{00}(x,y,z,t) = \frac{1}{L_x L_y L_z} + \sum_{n=1}^{\infty} F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t),
\]

where \( F_{ac} = \int_0^L c_n(u) \int_0^L c_n(v) \int_0^L f_{\phi}(u,v,w) dw \, dv \, du, \)

\[
e_{ac}(t) = \exp \left[ -\pi^2 n^2 D_{\phi}\left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) t \right];
\]

\[
C_{10}(x,y,z,t) = -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int s_n(u) \int c_n(v) \int g_L(u,v,w,T) \right.
\]

\[
\times e_{ac}(t) \int c_n(v) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{10}(u,v,w,\tau)}{\partial v} d w \, d v \, d u \, d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int s_n(u) \int c_n(v) \right.
\]

\[
\times e_{ac}(t) \int c_n(v) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{10}(u,v,w,\tau)}{\partial w} d w \, d v \, d u \, d \tau, \quad i \geq 1;
\]

\[
C_{01}(x,y,z,t) = -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int c_n(v) \int s_n(v) \int g_L(u,v,w,T) \right.
\]

\[
\times e_{ac}(t) \int c_n(v) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{01}(u,v,w,\tau)}{\partial v} d w \, d v \, d u \, d \tau \; - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int c_n(v) \right.
\]

\[
\times e_{ac}(t) \int c_n(v) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{01}(u,v,w,\tau)}{\partial w} d w \, d v \, d u \, d \tau;
\]

\[
C_{02}(x,y,z,t) = -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int s_n(u) \int c_n(v) \right.
\]

\[
\times e_{ac}(t) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{02}(u,v,w,\tau)}{\partial v} d w \, d v \, d u \, d \tau \; - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int c_n(v) \right.
\]

\[
\times e_{ac}(t) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{02}(u,v,w,\tau)}{\partial w} d w \, d v \, d u \, d \tau;
\]

\[
C_{03}(x,y,z,t) = -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int c_n(v) \int s_n(v) \right.
\]

\[
\times e_{ac}(t) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{03}(u,v,w,\tau)}{\partial v} d w \, d v \, d u \, d \tau \; - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac} c_n(x) c_n(y) c_n(z) e_{ac}(t) \left[ e_{ac}(-\tau) \int c_n(v) \right.
\]

\[
\times e_{ac}(t) \int s_n(v) g_L(u,v,w,T) \frac{\partial C_{03}(u,v,w,\tau)}{\partial w} d w \, d v \, d u \, d \tau.
\]
\[ \times \int_{0}^{l_{i}} s_{i}(w)C_{00}(u,v,w,\tau)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} \frac{\partial C_{00}(u,v,w,\tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n c_{n}(x) \times \]
\[ \times F_{ac} c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n c_{n}(x) \times \]
\[ \times \frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times n c_{n}(z) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times n c_{n}(z) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial v} d w d v d u d \tau ; \]
\[ C_{11}(x,y,z,t) = -\frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times g_{x}(u,v,w,T) \frac{\partial C_{10}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{10}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times n c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau \times \]
\[ \times n c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_{i}^{l_{i}} L_{i}^{l_{i}}} \sum_{n} n F_{ac} c_{n}(x) c_{n}(y) c_{n}(z) e_{ac}(t) \frac{\partial}{\partial \tau} s_{n}(w) C_{00}(u,v,w,\tau) \frac{\partial C_{00}(u,v,w,\tau)}{\partial u} d w d v d u d \tau - \]
\[-\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac}(x)c_n(x)c_n(y)e_{ac}(z)e_{ac}(t) e_{ac}(-\tau) s_n(w) \frac{C_{00}^T(u,v,w,\tau)}{P^T(u,v,w,\tau)} \times \ \]
\[\times \frac{\partial C_{10}(u,v,w,\tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n F_{ac}(x)c_n(x)c_n(y)e_{ac}(z)e_{ac}(t) e_{ac}(-\tau) s_n(w) \frac{C_{00}^T(u,v,w,\tau)}{P^T(u,v,w,\tau)} \times \ \]
\[\times F_{ac}(x)c_n(x)c_n(z)e_{ac}(t) e_{ac}(-\tau) c_n(w) s_n(v) c_n(w) \frac{C_{00}^T(u,v,w,\tau)}{P^T(u,v,w,\tau)} \times \ \]
\[\times F_{ac}(x)c_n(x)c_n(z)e_{ac}(t) e_{ac}(-\tau) s_n(w) C_{10}(u,v,w,\tau) \frac{\partial C_{10}(u,v,w,\tau)}{\partial w} d w d v d u d \tau. \]

REFERENCES


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