INVESTIGATION OF NONLINEAR DYNAMICS IN THE BOOST CONVERTER: EFFECT OF CAPACITANCE VARIATIONS

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ABSTRACT

The electronic domain is highly nonlinear, hence it is valuable to study the nonlinear effect particularly the chaos and bifurcation. The present paper deals with the simulative analysis of nonlinear dynamics in the boost converter with the help of bifurcation diagrams. In this brief communication, the current through inductor (I_L) was considered as state variable and reference current (I_{REF}) was considered as a controlled variable. The capacitor value was varied from 1\textmu f to 50\textmu f while the other parameter was kept unaltered. It was observed that, as the value of capacitor was increased, the corresponding period-1 bifurcation, period-2 bifurcations, and period-3 bifurcations points were shifted in incremental order. It was also observed that period-2 bifurcations, and period-3 bifurcations points were vanished with an increasing capacitor value (C) and supply voltage (V_{IN}).

KEYWORDS: BIFURCATION, BOOST CONVERTER, CHAOS, NONLINEAR EFFECT

1. INTRODUCTION

The nonlinear dynamics has many applications and hence it is widely investigated in many fields of science and engineering domains. The nonlinearity depends on all state variables of physical system. The nonlinearity produces a chaos which is due to initial condition problems and sensitivity to these conditions. The first kind of nonlinearity and particularly chaotic effect was observed to the Henri Poincaré on celestial mechanics around 1900 [1]. In 1963 Lorenz gave an idea that simple nonlinear systems can have complex, chaotic behaviour in his seminal research paper ‘Deterministic Non-periodic Flow’ [1]. The electronics’ field is very dynamic hence it is valuable to study nonlinear effect for better understanding of the subject.

The Van der Pol first of all shows that the nonlinear dynamic behaviours in the field of electronics [2]. Soon after, many researchers found out the nonlinear effect such as bifurcation and chaos in many electronic circuits topologies. The power converters got a lot of attention between all of them. In 1980, Ballieul et al gave the idea about chaotic behaviour in DC-DC converter [3]. Parui et al give the idea of bifurcations in the boost converter [4]. Deane et al emphasized their studies on instability, sub-harmonics and chaos in power electronic systems [5]. Chan et al showed the quasi-periodicity to period-doubling bifurcations in the boost converter [6]. Dongale et al shows the chaotic behaviour of boost converter using bifurcation diagram [7]. Lu et al studied the bifurcation phenomena in parallel-connected boost converter system [8]. L. Chua’s et al work on nonlinear circuits (Chua’s Circuit) based on capacitor and diode which is very

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famous in electronics today [9]. The Chua’s diode is now famous with the name ‘Memristor’ [10-14]. The boost converter is also known as step up converter whose primary aim is to increase the average output voltage higher than input voltage or supply voltage [15-17]. The present paper investigated the chaotic effect in the boost converter with the help of bifurcation diagram. This work is aims to study the stability of the system using bifurcation diagram. Rest of paper is portrayed as follows, after brief introduction in the first section, the second section deals with introduction of chaos and bifurcation. Third section describes the working of boost converter. The fourth section deals with the chaos and bifurcation analysis. At the end results are reported.

2. NONLINEAR DYNAMICS

The study of nonlinear dynamics has come into game when Newton invented the differential equations. He solved the classical two body problems with the help of differential equations, but the classical three body problems cannot be solved with the help of Newton’s numerical method. The breakthrough came when Poincaré found that celestial mechanics for highly nonlinear systems [18].

The Bifurcation theory was introduced by Henri Poincare and chaos theory was proposed by Edward Lorenz. The chaos is occurs due to the initial condition problem and discreteness of state variables. The chaos is a very robust phenomenon and a few years ago it was treated as only noise. The major breakthrough came when Edward Lorenz model the weather and formulate the Butterfly Effect. The butterfly effect says that if we make small change in any of the state variable of system then it is impossible to predict the final outcome of system i.e. small variation can become a large diversion [19]. Bifurcation phenomenon has two types, first is smooth bifurcation phenomenon and other is border collision bifurcation phenomenon [7, 20].

Each of the bifurcations may give rise to a distinct route to chaos if the bifurcations appear repeatedly upon changing the bifurcation parameter [21]. In the state control space, the place at which bifurcations occur are called bifurcation points [19]. Many times, it is possible to predict the system stability with the help of equilibrium, periodic, quasi-periodic, and chaotic behaviors of bifurcations diagrams. The present paper was aimed to find out system stability with the help of bifurcations diagrams. In the present study, bifurcation diagram for Inductor Current (I_L) with reference current (I_{REF}) are plotted. The inductor (I_L) current worked as a state variable and the reference current (I_{REF}) is worked as a controlled variable [7]. The effects on stability by means of variation of capacitor value are depicted for each case. The period-1, 2 and 3 bifurcations points are shown by solid lines in the figure 2 to 5 respectively. The linear and steady-state performance of converters and electric controls were investigated by conventional means [22-31], but nonlinear aspects and dynamic behaviour of converter was not studied and examined at its large.

3. BOOST CONVERTER

The boost converter regulates the average output voltage to higher than input voltage or supply voltage hence it is known as step up DC-DC converter or DC amplifier. The typical DC to DC boost converter is shown in fig. 1. The output voltage of boost converter is controlled by controlling the duty cycle of MOSFET. The boost converter increases the magnitude of output voltage by using energy storing principle in the inductor and capacitor. These lumped circuit components are responsible for the nonlinear dynamic behaviour in the boost converter. The detailed working of boost converter can be found in the reference [7].
Figure 1: The DC-DC Boost converter. The circuit diagram consist of DC supply ($V_{\text{IN}}$), Inductor in series (L), MOSFET as a switch, Diode (D), Capacitor (C) and Load resistance ($R_L$) [7].

The state dynamics or equations of boost converter are given as, [32-33]

\[
\begin{align*}
\frac{dV}{dt} &= -\frac{1}{RC} V_0 \\
\frac{di}{dt} &= -\frac{1}{L} V_{\text{in}} \\
\frac{dV}{dt} &= -\frac{1}{RC} V_0 + \frac{1}{C} i \\
\frac{di}{dt} &= -\frac{1}{L} V_0 + \frac{1}{L} V_{\text{in}}
\end{align*}
\]

(For $nT \leq t < (n + d) T$) ................................ (1)

(For $(n + d) T \leq t < (n + 1) T$) .................. (2)

Where, $V_0$ is the output voltage, $V_{\text{IN}}$ supply voltage, $d$ is the duty cycle and $n$ is an integer.

4. INVESTIGATIONS OF CHAOTIC AND BIFURCATION PHENOMENA IN BOOST CONVERTER

The nonlinear dynamic of boost converter was observed using bifurcation diagram. The following figures (2-5) show the variation of capacitor (C) as well as supply voltage ($V_{\text{IN}}$) and corresponding effect on bifurcations diagrams of single stage boost converter. The value of capacitor (C) is varied from 1 $\mu$F to 50 $\mu$F and the value of supply voltage ($V_{\text{IN}}$) is varied from 5V to 20V. The figure 2 (A, B, C and D) is for supply voltage ($V_{\text{IN}}$) 5V and remaining figure 3, 4 and 5 shows the performance of boost converter at supply voltage ($V_{\text{IN}}$) 10V, 15V and 20V respectively. For this simulation series inductor considered as 2 mH, load resistance ($R_L$) considered as 20 $\Omega$, and switching frequency is considered as 10 KHz.
Figure 2 (A, B, C, D, E and F): Bifurcation diagram for Inductor Current ($I_L$) with reference current ($I_{REF}$). Here the capacitor (C) is varied and other parameters are kept fixed. The figures A, B, C, D, E and F represent the variations in the capacitor value viz. A= 1\mu f, B= 10\mu f, C= 20\mu f, D= 30\mu f, E= 40\mu f and F= 50\mu f respectively at supply voltage ($V_{IN}$) equals to 5V. The red solid line A, B and C represent the period- 1, 2 and period- 3 bifurcation respectively.
Figure 3 (A, B, C, D, E and F): Bifurcation diagram for Inductor Current ($I_L$) with reference current ($I_{REF}$). Here the capacitor (C) is varied and other parameters are kept fixed. The figures A, B, C, D, E and F represent the variations in the capacitor value viz. A= $1\mu$f, B= $10\mu$f, C= $20\mu$f, D= $30\mu$f, E= $40\mu$f and F= $50\mu$f respectively at supply voltage ($V_{IN}$) equals to 10V. The blue solid line A, B and C represent the period- 1, 2 and period- 3 bifurcation respectively.
Figure 4 (A, B, C, D, E and F): Bifurcation diagram for Inductor Current ($I_L$) with reference current ($I_{REF}$). Here the capacitor (C) is varied and other parameters are kept fixed. The figures A, B, C, D, E and F represent the variations in the capacitor value viz. A= 1µf, B= 10µf, C= 20µf, D= 30µf, E= 40µf and F= 50µf respectively at supply voltage ($V_{IN}$) equals to 15V. The black solid line A, B and C represent the period- 1, 2 and period- 3 bifurcation respectively.
Figure 5 (A, B, C, D, E and F): Bifurcation diagram for Inductor Current ($I_L$) with reference current ($I_{REF}$). Here the capacitor (C) is varied and other parameters are kept fixed. The figures A, B, C, D, E and F represent the variations in the capacitor value viz. A= 1µf, B= 10µf, C= 20µf, D= 30µf, E= 40µf and F= 50µf respectively at supply voltage ($V_{IN}$) equals to 20V. The red solid line A, B and C represent the period- 1, 2 and period- 3 bifurcation respectively.
Table 2: Relation between Period-Bifurcation (1, 2 and 3), State Variable and Controlled Variable with Variations of Capacitor Value (1µf to 50µf). The Blank Cell Indicate the Absence of Period-Bifurcation.

<table>
<thead>
<tr>
<th>Capacitor Value (µf)</th>
<th>Period- Bifurcation</th>
<th>Supply Voltage (V_{IN}) = 5V</th>
<th>Supply Voltage (V_{IN}) = 10V</th>
<th>Supply Voltage (V_{IN}) = 15V</th>
<th>Supply Voltage (V_{IN}) = 20V</th>
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<tr>
<td></td>
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<td>I_{L} (A)</td>
<td>I_{REF} (A)</td>
<td>I_{L} (A)</td>
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<td></td>
<td>Period- 2 Bifurcation</td>
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<td>0.90</td>
<td>1.85</td>
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<tr>
<td></td>
<td>Period- 3 Bifurcation</td>
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<td>2.10</td>
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<tr>
<td>10 µf</td>
<td>Period- 1 Bifurcation</td>
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<tr>
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5. RESULT AND DISCUSSION

In this study, the chaotic effect of boost converter was investigated by means of bifurcation diagram. Table 2 represents the relationship between period-bifurcation and corresponding effect on controlled variable $I_{\text{REF}}$ and state variable $I_L$ with variations of capacitor value (1 $\mu$F to 50 $\mu$F) and supply voltage ($V_{\text{IN}}$ = 5V to 20V). Over the period-1 bifurcation point, inductor current ($I_L$) having single unique value for turn on the switch. For period-2 bifurcations there are two values of inductor current ($I_L$) to turn on the switch. As we go further, the system undergoes stable period-3 operation, and eventually becomes chaotic.

It is clearly seen from fig. 2 to fig. 5 and table 2, that single stage boost converter degrades its stability at low voltage and at low value of capacitor. It is clearly shown from fig. 2-5 that bifurcation points are shifted in increment order as we increase the value of capacitor and it become chaotic after period-3 bifurcations in each case. The simulation results of boost converter also showed that the existence of only one stable point at higher value of capacitor (C) and higher valve of supply voltage ($V_{\text{IN}}$). The period-3 bifurcation vanishes from 20 $\mu$F and the period-2 bifurcation vanishes from 30 $\mu$F. In this case, region above the period-3 bifurcation is a chaotic region for the boost converter.

REFERENCES