EXTENDED PROCESS ALGEBRA FOR COST ANALYSIS

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ABSTRACT

Recently, the computational cost in execution of processes is considered to be one of the important factors in concurrent systems and is beginning to interest to us. For example, the denial-of-service attacks are related to it closely.Several formal systems have been proposed for reasoning about concurrent systems. The Algebra of Communicating Processes (ACP) is proposed by J. A. Bergstra and J. W. Klop, and an algebraic system for such reasoning. In such formal systems, the equivalence between processes is the fundamental relation for reasoning, and several kinds of equivalence have been investigated in the process algebras. These equivalence relations are based on behavior of processes. We developed a formal system for reasoning about computational costs of concurrent systems in the previous works, based on the Milner's pi-calculus. In this paper, we investigate another system based on a subsystem of ACP, BPA(Basic Process Algebra). We first introduce cost value equality, which is based on operational semantics of the process algebra and then give its formalized equality, called formal cost equality. We study theoretical properties on the two equalities, such as soundness and completeness. Moreover, we show some results on the cost formalization in other subsystems of ACP, such as PAP and BPA+δ.

KEYWORDS

Process algebra, the algebra of communicating systems, ACP, cost analysis, concurrent systems.

1. INTRODUCTION

A denial-of-service (DoS) attack is an attempt to make a system resource unavailable to its intended users. Several types of DoS attacks are known, and those that are caused by the vulnerability of a network protocol are known to be serious. A SYN flood attack is a typical example of this, exploiting the vulnerability of TCP's three-way handshake with respect to inappropriate imbalances between the computational costs of clients and server.

Meadow's work [1] is a pioneering study of a formal framework for describing and analyzing protocols with respect to DoS attack resistance. The framework is a kind of protocol description in the Alice-and-Bob style, in which a computational cost is annotated with each communication step of the protocol. Although the Alice-and-Bob-style specification is easy to understand, it lacks accuracy in some cases. To address these inaccuracies, we propose a formal system for analyzing DoS attack resistance. The system is called the spice-calculus [2] and is based on Milner's pi-calculus [3] and Abadi and Gordon's spi-calculus [4]. Because a process calculus is more accretive and more precise than the Alice-and-Bob style for describing communication processes in protocols, the spice-calculus enables grasping the dynamism of a protocol. Consequently, it clarifies the cost imbalances between clients and servers.

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In the spice calculus, we know the cost during processes' execution through the annotation which attached to the transition which formulates execution of a process. We therefore say that the approach to the computational cost in the spice-calculus is indirect. The purpose of this paper is to develop an algebraic framework for cost analysis, which gives us more direct approach than the previous one. In order to give an algebraic framework, we adopt another formal system, called ACP, in place of the pi- and spi-calculus.

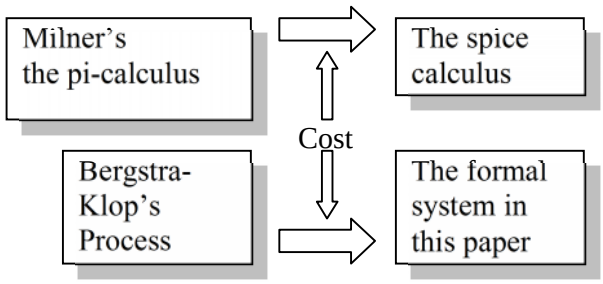

Figure 1.Overview of our works.

The Algebra of Communicating Processes, ACP, is an algebraic approach to reasoning about concurrent systems, which was proposed by Jan Bergstra and Jan Willem Klop [5][6]. In contrast to CSP, CCS and the pi-calculus, the development of ACP focus on the algebra of processes. More concretely, the equation system is built up on the basis of the transition relation between processes. We construct a framework for cost analysis, giving a cost equivalence relation between processes to ACP.

In Section 2, we give a brief introduction to the Bergstra-Klop-style process algebra. In Section 3, we formalize the notion of costin the process algebra. We introduce cost value equality based on the equational semantics of the process algebra. The equality is defined on a subsystem BPA of ACP. In Section 4, we introduce an equational axiomatic system to the cost value equality, which is called formal cost equality. We investigate the theoretical properties between the cost value equality and the formal cost equality, such as soundness and completeness.

2. FRAGMENTS OF BERGSTRA-KLOP'S CALCULI

In this section, we show a brief introduction on the process algebra ACP proposed by Bergstra and Klop, and its subsystems [6,7]. The expressions, called *terms*, of ACP are defined as follows.

Definition 1 (Terms of ACP) Terms of ACP are defined inductively by the following rules.

- *Atomic actions* $a, b, c, \ldots$ are terms;
- The *silent action* τ is a term;
- An *alternative composition* $s + t$ of terms s and s and t is a term;
- A *sequential composition* $s \cdot t$ of terms s and s and t is a term;
- A *merge* $s \parallel t$ of terms s and s and t is a term;
- A *left merge* $s \lfloor\!\lfloor t$ of terms s and s and t is a term;
- A *communication* $s|t$ of terms s and s and t is a term;
- An *encapsulation* $\partial_H(t)$ of a term t is a term, where H is a set of atomic actions.

The axioms of ACP are given as a system of equation. We often consider two subsystems of ACP, BPA(Basic ProcessAlgebra) and PAP (Process Algebra of Parallelism). The axioms of BPA are given as follows.

Definition 2 (Axioms of BPA) Axioms of BPA are given by the following equations.

- $x + y = y + x$ (A1)
- $(x + y) + z = x + (y + z)$ (A2)
- $x + x = x$ (A3)
- $(x + y) \cdot z = x \cdot z + y \cdot z$ (A4)
- $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ (A5)

Actually, this equivalence relation coincides with the bi-simulation relation derived from the transition relation defined in the followings.

Definition 3 (Transition relation of BPA) Transition relation of BPA is defined inductively by the following rules.

- $v \rightarrow^{v} \checkmark$
- if $x \rightarrow^{v} \checkmark$, then $x + y \rightarrow^{v} \checkmark$,
- if $x \rightarrow^{v} x'$, then $x + y \rightarrow^{v} x'$,
- if $y \rightarrow^{v} \checkmark$, then $x + y \rightarrow^{v} \checkmark$,
- if $y \rightarrow^{v} y'$, then $x + y \rightarrow^{v} y'$,
- if $x \rightarrow^{v} \checkmark$, then $x \cdot y \rightarrow^{v} y$, and
- if $x \rightarrow^{v} x'$, then $x \cdot y \rightarrow^{v} x' \cdot y$.

where x, y, z run over the set of processes.

Definition 4 (Axioms of PAP) The axioms of PAP are A1-A5 of BPA and the following rules

- $x \cdot y = (x \,\|\!\lfloor\, y + y \,\|\!\lfloor\, x) + x|y$ (M1)
- $v \,\|\!\lfloor\, y = v \cdot y$ (LM2)
- $(v \cdot x) \cdot y = v \cdot (x \cdot y)$ (LM3)
- $(x + y) \,\|\!\lfloor\, z = x \,\|\!\lfloor\, z + y \,\|\!\lfloor\, z$ (LM4)
- $v \mid w = \gamma(v, w)$ (CM5)
- $v \mid (w \cdot y) = \gamma(v, w) \cdot y$ (CM6)
- $(v \cdot x) \mid w = \gamma(v, w) \cdot x$ (CM7)
- $(v \cdot x) \mid (w \cdot y) = \gamma(v, w) \cdot (x \cdot y)$ (CM8)
- $(x + y)|z = x|z + y|z$ (CM9)
- $x \mid (y + z) = x|y + x|z$ (CM10)

These equations are also derived from the translation rules of PAP:

Definition 5 (Transition relation of PAP) Transition relation of PAP is defined inductively by the translation rules of BPA and the following rules.

- if $x \rightarrow^{v} \checkmark$ and $x \cdot y \rightarrow^{v} \checkmark$,
- if $x \rightarrow^{v} x'$ then $x \cdot y \rightarrow^{v} x' \cdot y$,
- if $y \rightarrow^{v} \checkmark$ and $x \cdot y \rightarrow^{v} \checkmark$,
- if $y \rightarrow^{v} y'$ then $x \cdot y \rightarrow^{v} x \cdot y'$,

- if $x \rightarrow^{v} \checkmark$ and $y \rightarrow^{w} \checkmark$, then $x \cdot y \rightarrow^{\gamma(v,w)} \checkmark$,
- if $x \rightarrow^{v} \checkmark$ and $y \rightarrow^{w} y'$, then $x \cdot y \rightarrow^{\gamma(v,w)} y'$,

- if $x \rightarrow^{v} x'$ and $y \rightarrow^{w} \checkmark$, then $x \square y \rightarrow^{\gamma(v,w)} x'$,
- if $x \rightarrow^{v} x'$ and $y \rightarrow^{w} y'$, then $x \square y \rightarrow^{\gamma(v,w)} x' \square y'$,

- if $x \rightarrow^{v} \checkmark$ and $x \| y \rightarrow^{v} y$,
- if $x \rightarrow^{v} x'$ and $x \| y \rightarrow^{v} x' \square y$,

- if $x \rightarrow^{v} \checkmark$ and $y \rightarrow^{w} \checkmark$, then $x|y \rightarrow^{\gamma(v,w)} \checkmark$,
- if $x \rightarrow^{v} \checkmark$ and $y \rightarrow^{w} y'$, then $x|y \rightarrow^{\gamma(v,w)} y'$,
- if $x \rightarrow^{v} x'$ and $y \rightarrow^{w} \checkmark$, then $x|y \rightarrow^{\gamma(v,w)} x'$,
- if $x \rightarrow^{v} x'$ and $y \rightarrow^{w} y'$, then $x|y \rightarrow^{\gamma(v,w)} x' \square y'$,

Definition 6 (Bisimulation relation) Two processes s and t are *bisimular*, denoted $s \underline{\leftrightarrow} t$,if s and t satisfies the relation (s R t) defined by the following rules.

- if $(s\ R\ t)$ and $s \rightarrow^{v} s'$, then $t \rightarrow^{v} t'$ and $(s' R\ t')$.
- if $(s\ R\ t)$ and $t \rightarrow^{v} t'$, then $s \rightarrow^{v} s'$ and $(s' R\ t')$.
- if $(s\ R\ t)$ and $s \rightarrow^{v} \checkmark$, then $s \rightarrow^{v} \checkmark$,
- if$(s\ R\ t)$ and $t \rightarrow^{v} \checkmark$, then $t \rightarrow^{v} \checkmark$.

As mentioned above, both in BPA and in PAP, the bisimulation relation derived from each transition is equivalent to the equivalence relation defined by the axiom, system respectively.

Proposition 7 (Soundness of axioms with respect to bisimulation)[6,7]For terms s and t, if $s = t$, then $s \underline{\leftrightarrow} t$, both in BPA and in PAP.

Theorem 8 (Completeness of axioms with respect to bisimulation)[6,7] For terms s and t, if $s \underline{\leftrightarrow} t$, then $s = t$, both in BPA and in PAP.

In this paper, we develop formalization of costs focusing on these two sub-systems, BPA and PAP. Hence, we would like to omit details of the full fragment of ACP for lack of space.

3. SEMANTIC FORMALIZATION OF COSTS IN BPA

First, we define the notion of cost in the process algebra in the style of Bergstra-Klop.

Definition 9 (Cost Summand) A*summand* is a set of multisets of actions.A *cost summand*$\mathrm{cost}(s)$of a process s is the set of multisets of actions defined as

$$\mathrm{cost}(s) = \{a_{i,1} + a_{i,2} + \cdots + a_{i,n_i} \mid i = 1, \ldots, m\},$$

where the transition paths starting at s are assumed to be

$$s \rightarrow^{a_{1,1}} s_{1,1} \rightarrow^{a_{1,2}} s_{1,2} \rightarrow \cdots \rightarrow s_{1,n_1-1} \rightarrow^{a_{1,n_1}} \checkmark,$$
$$s \rightarrow^{a_{2,1}} s_{2,1} \rightarrow^{a_{2,2}} s_{2,2} \rightarrow \cdots \rightarrow s_{2,n_2-1} \rightarrow^{a_{2,n_2}} \checkmark,$$
$$\cdots$$
$$s \rightarrow^{a_{m,1}} s_{m,1} \rightarrow^{a_{m,2}} s_{m,2} \rightarrow \cdots \rightarrow s_{m,n_m-1} \rightarrow^{a_{m,n_m}} \checkmark,$$

where $s_{i,n_i-1} = a_{i,n_i-1}$ $(i = 1, \ldots, m)$.
The expression $a_{i,1} + a_{i,2} + \cdots + a_{i,n_i}$ denotes the multiset whose elements are $a_{i,1}, a_{i,2}, \ldots, a_{i,n_i}$.

Example 10. Consider a process

$$P = \text{open} \cdot (\text{read} + \text{write}) \cdot \text{close}$$

whereopen, read and write are actions. Then its summand $\text{cost}(P)$ is

$$\{\text{open} + \text{read} + \text{close}, \text{open} + \text{write} + \text{close}\}.$$

Informally speaking, we may regard the cost summand $\text{cost}(P)$ as a combination of the possible costs.

Definition 11 (Cost Value Equality) The *cost value equality* between processes, denote s ≈ t, is defined as cost(s) = cost(t).

Example. Terms$\text{open} \cdot (\text{read} + \text{write}) \cdot \text{close}$, $\text{open} \cdot (\text{read} \cdot \text{close} + \text{write} \cdot \text{close})$ and $(\text{open} \cdot \text{read} \cdot \text{close} + \text{open} \cdot \text{write} \cdot \text{close})$ have the same cost summand$\{\text{open} + \text{read} + \text{close}, \text{open} + \text{write} + \text{close}\}$. Hence, $\text{open} \cdot (\text{read} + \text{write}) \cdot \text{close} \approx \text{open} \cdot (\text{read} \cdot \text{close} + \text{write} \cdot \text{close}) \approx (\text{open} \cdot \text{read} \cdot \text{close} + \text{open} \cdot \text{write} \cdot \text{close})$.

We next present basic properties on the cost value equality.In the fragment of BPA, we have the following properties.

Proposition 12. The cost value equality is an equivalence relation on processes in BPA.

Proof. Reflexivity, symmetry and transitivity are trivially derived from the ones of the equality between sets.

Proposition 13.(Congruence of the Cost Value Equality in BPA) The cost value equality is a congruence relation on processes in BPA, that is, if$s \approx t$ then $s[s] \approx u[t]$, where u[ ] means a process with a hole [ ]and $u[s]$ a process which is obtained by replacing a hole [ ] in the u [ ] by s.

Proof.This proposition is derived from the following properties **Lemma 14** and **Lemma 15.**

Lemma 14.For terms of s, s', t, t', if $s \approx s'$ and $t \approx t'$ then $s + t \approx s' + t'$.
Lemma 15. If $s \approx s'$ and $t \approx t'$then $s \cdot t \approx s' \cdot t'$.

Proof of Lemma 14.

Supposethat $s \approx s'$, $t \approx t'$, and that we have the transition paths starting at s, s', t, t' such as

$$s \xrightarrow{a_{i,1}} s_{i,1} \xrightarrow{a_{i,2}} s_{i,2} \rightarrow \cdots \xrightarrow{a_{i,n_i-1}} s_{i,n_i-1} \xrightarrow{a_{i,n_i}} \checkmark (i = 1, \ldots, N)$$

$$s' \xrightarrow{a'_{i,1}} s'_{i,1} \xrightarrow{a'_{i,2}} s'_{i,2} \rightarrow \cdots \xrightarrow{a'_{i,n'_i-1}} s'_{i,n'_i-1} \xrightarrow{a'_{i,n'_i}} \checkmark (i = 1, \ldots, N')$$

$$t \xrightarrow{b_{i,1}} t_{i,1} \xrightarrow{b_{i,2}} t_{i,2} \rightarrow \cdots \xrightarrow{b_{i,m_i-1}} t_{i,m_i-1} \xrightarrow{b_{i,m_i}} \checkmark (i = 1, \ldots, M)$$

$$t' \xrightarrow{b'_{i,1}} t'_{i,1} \xrightarrow{b'_{i,2}} t'_{i,2} \rightarrow \cdots \xrightarrow{b'_{i,m_i-1}} t'_{i,m_i-1} \xrightarrow{b'_{i,m_i}} \checkmark (i = 1, \ldots, M')$$

We let $s_{i,n_i-1}, s'_{i,n'_i-1}, t_{i,m_i-1}, t'_{i,m_i-1}$be a_{i,n_i}, a'_{i,n'_i}, b_{i,m_i}, b'_{i,m_i}, respectively.

From these transition paths, we know that

$$\mathrm{cost}(s) = \bigcup_{i=1}^{N}\{\sum_{j=1}^{n_i} a_{ij}\}, \qquad \mathrm{cost}(s') = \bigcup_{i=1}^{N'}\{\sum_{j=1}^{n'_i} a'_{ij}\},$$

$$\mathrm{cost}(t) = \bigcup_{i=1}^{M}\{\sum_{j=1}^{m_i} b_{ij}\}, \qquad \mathrm{cost}(t') = \bigcup_{i=1}^{M'}\{\sum_{j=1}^{m'_i} b'_{ij}\},$$

Term $s + t$ can have the following transitions

$$s + t \xrightarrow{a_{i,1}} s_{i,1} \xrightarrow{a_{i,2}} s_{i,2} \rightarrow \cdots \xrightarrow{a_{i,n_i-1}} s_{i,n_i-1} \xrightarrow{a_{i,n_i}} \checkmark (i = 1, \ldots, N)$$

$$s + t \xrightarrow{b_{i,1}} t_{i,1} \xrightarrow{b_{i,2}} t_{i,2} \rightarrow \cdots \xrightarrow{b_{i,m_i-1}} t_{i,m_i-1} \xrightarrow{b_{i,m_i}} \checkmark (i = 1, \ldots, M)$$

according to the third and fifth rules of **Definition 3**. Therefore,

$$\mathrm{cost}(s + t) = \mathrm{cost}(s) \square \mathrm{cost}(t).$$

Similarly, $\mathrm{cost}(s' + t') = \mathrm{cost}(s') \square \mathrm{cost}(t')$. By the assumption that $s \approx s'$ and $t \approx t'$, we have $\mathrm{cost}(s) = \mathrm{cost}(s')$ and $\mathrm{cost}(t) = \mathrm{cost}(t')$. Hence, $\mathrm{cost}(s + t) = \mathrm{cost}(s' + t')$.

Q.E.D.

Proof of Lemma 15.

We make an assumption similar to the above proof. According to the sixth and seventh rules of Definition 3, we know that the term $s \square t$ can make the following transitions

$$s \square t \xrightarrow{a_{i,1}} s_{i,1} \square t \xrightarrow{a_{i,2}} s_{i,2} \square t \rightarrow \cdots \xrightarrow{a_{i,n_i-1}} s_{i,n_i-1} \square t \xrightarrow{a_{i,n_i}}$$
$$t \xrightarrow{b_{j,1}} t_{j,1} \xrightarrow{b_{j,2}} t_{j,2} \rightarrow \cdots \xrightarrow{b_{j,m_j-1}} t_{j,m_j-1} \xrightarrow{b_{j,m_j}} \checkmark$$
$$(i = 1, \ldots, N, j = 1, \ldots, M)$$

Therefore,

$$\mathrm{cost}(s \square t) = \bigcup_{i=1}^{N}\bigcup_{j=1}^{M}\left\{\sum_{k=1}^{n_i} a_{ik} + \sum_{k=1}^{m_j} b_{jk}\right\}$$

Similarly, we have

$$\mathrm{cost}(s' \square t') = \bigcup_{i=1}^{N'}\bigcup_{j=1}^{M'}\left\{\sum_{k=1}^{n_i} a'_{ik} + \sum_{k=1}^{m_j} b'_{jk}\right\}$$

By the assumption that $s \approx s'$ and $t \approx t'$,

$$\mathrm{cost}(s) = \bigcup_{i=1}^{N}\left\{\sum_{j=1}^{n_i} a_{ij}\right\} = \bigcup_{i=1}^{N'}\left\{\sum_{j=1}^{n'_i} a'_{ij}\right\} = cost(s')$$

and

$$\mathrm{cost}(t) = \bigcup_{i=1}^{M}\left\{\sum_{j=1}^{m_i} b_{ij}\right\} = \bigcup_{i=1}^{M'}\left\{\sum_{j=1}^{m'_i} b'_{ij}\right\} = \mathrm{cost}(t')$$

Hence, we know $\mathrm{cost}(s \square t) = \mathrm{cost}(s' \square t')$.

Q.E.D.

4. Axiomatic Formalization of Costs in BPA

In this section, we give axioms to the cost value equality as an equation system. The equality defined by the equation system is called the formal cost equality. We study the theoretical relationship between the cost value equality and the formal cost equality.

The cost value equality is axiomatized by the following equality rules.

Definition 16(Formal cost equality in BPA) We call the relation $s \sim t$ defined above the formal cost equality.

- $x + y \sim y + x$, (A'1)
- $(x + y) + z \sim x + (y + z)$, (A'2)
- $x + x \sim x$, (A'3)
- $(x + y) \cdot z \sim x \cdot z + y \cdot z$, (A'4)
- $(x \cdot y) \cdot z \sim x \cdot (y \cdot z)$, (A'5)
- $x \cdot y \sim y \cdot x$. (A'6)

The noticeable difference of the formal cost equality from the equivalence relation $s = t$ defined in the previous section is the rule A'6.
The relation $s \sim t$ satisfies soundness and completeness with respect to the cost value equality $s \approx t$.

Example.A term $\mathrm{open} \square (\mathrm{read} + \mathrm{write}) \square \mathrm{close}$ is equivalent to $\mathrm{open} \square (\mathrm{read} \square \mathrm{close} + \mathrm{write} \square \mathrm{close})$ but not to $(\mathrm{open} \square \mathrm{read} \square \mathrm{close} + \mathrm{open} \square \mathrm{write} \square \mathrm{close})$, according to Definition 4. On the other hand, $\mathrm{open} \square (\mathrm{read} + \mathrm{write}) \square \mathrm{close} \square \mathrm{open} \square (\mathrm{read} \square \mathrm{close} + \mathrm{write} \square \mathrm{close}) \square (\mathrm{open} \square \mathrm{read} \square \mathrm{close} + \mathrm{open} \square \mathrm{write} \square \mathrm{close})$.

Theorem 17 The formal cost equality $s \sim t$ is *sound* with respect to the cost value equality $s \approx t$, in other words, $s \sim t \rightarrow s \approx t$ for terms s and t. Especially, $s = t \rightarrow s \approx t$.

Proof Since we already know that the cost value equality is an equivalence and congruence relation from Proposition 12 and 13, it is enough to show that equalities (A'1),…,(A'6) holds with respect to the cost value equality.

Case of (A'1). Assume that the transition paths of s and t are

$$s \rightarrow^{a_{i,1}} s_{i,1} \rightarrow^{a_{i,2}} s_{i,2} \rightarrow \cdots \rightarrow^{a_{i,n_i-1}} s_{i,n_i-1} \rightarrow^{s_{i,n_i-1}} \checkmark (i = 1, \ldots, N) \text{and}$$
$$t \rightarrow^{b_{i,1}} t_{i,1} \rightarrow^{b_{i,2}} t_{i,2} \rightarrow \cdots \rightarrow^{b_{i,m_i-1}} t_{i,m_i-1} \rightarrow^{t_{i,m_i-1}} \checkmark (i = 1, \ldots, M),$$

respectively. We let s_{i,n_i-1},t_{i,m_i-1} be a_{i,n_i}, b_{i,m_i}, respectively.

We then obtained that $\mathrm{cost}(s) = \bigcup_{i=1}^{N}\{\sum_{j=1}^{n_i} a_{ij}\}$and $\mathrm{cost}(t) = \bigcup_{i=1}^{M}\{\sum_{j=1}^{m_i} b_{ij}\}$. By the third and fifth rules of Definition 3, we can express the transition paths starting from s + t as

$$s + t \to^{a_{i,1}} s_{i,1} \to^{a_{i,2}} s_{i,2} \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \to \checkmark (i = 1, \ldots, N)$$
$$s + t \to^{b_{i,1}} t_{i,1} \to^{b_{i,2}} t_{i,2} \to \cdots \to^{b_{i,m_i-1}} t_{i,m_i-1} \to \checkmark (i = 1, \ldots, M)$$

Therefore we have $\mathrm{cost}(s + t) = \mathrm{cost}(s) \square \mathrm{cost}(t)$. Similarly, we also have $\mathrm{cost}(t + s) = \mathrm{cost}(t) \square \mathrm{cost}(s)$. Hence $\mathrm{cost}(s + t) = \mathrm{cost}(t + s)$.

Case of (A'2). Assume that the transition paths starting froms, t and u are

$$s \to^{a_{i,1}} s_{i,1} \to^{a_{i,2}} s_{i,2} \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \to^{s_{i,n_i-1}} \checkmark (i = 1, \ldots, N),$$
$$t \to^{b_{i,1}} t_{i,1} \to^{b_{i,2}} t_{i,2} \to \cdots \to^{b_{i,m_i-1}} t_{i,m_i-1} \to^{t_{i,m_i-1}} \checkmark (i = 1, \ldots, M), \text{ and}$$
$$u \to^{c_{i,1}} u_{i,1} \to^{c_{i,2}} u_{i,2} \to \cdots \to^{c_{i,l_i-1}} u_{i,l_i-1} \to^{u_{i,l_i-1}} \checkmark (i = 1, \ldots, L)$$

respectively. Then the transition paths starting from $(s + t) + u$ are

$$(s + t) + u \to^{a_{i,1}} s_{i,1} \to^{a_{i,2}} s_{i,2} \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \to^{s_{i,n_i-1}} \checkmark (i = 1, \ldots, N),$$
$$(s + t) + u \to^{b_{i,1}} t_{i,1} \to^{b_{i,2}} t_{i,2} \to \cdots \to^{b_{i,m_i-1}} t_{i,m_i-1} \to^{t_{i,m_i-1}} \checkmark (i = 1, \ldots, M), \text{ and}$$
$$(s + t) + u \to^{c_{i,1}} u_{i,1} \to^{c_{i,2}} u_{i,2} \to \cdots \to^{c_{i,l_i-1}} u_{i,l_i-1} \to^{u_{i,l_i-1}} \checkmark (i = 1, \ldots, L)$$

We therefore know that

$$\mathrm{cost}\big((s + t) + u\big) = \bigcup_{i=1}^{N}\{\sum_{j=1}^{n_i} a_{ij}\} \square \bigcup_{i=1}^{M}\left\{\sum_{j=1}^{m_i} b_{ij}\right\} \cup \bigcup_{i=1}^{L}\{\sum_{j=1}^{l_i} c_{ij}\}$$
$$= \mathrm{cost}(s) \square \mathrm{cost}(t) \square \mathrm{cost}(u).$$

Similarly, we have $\mathrm{cost}\big(s + (t + u)\big) = \mathrm{cost}(s) \square \mathrm{cost}(t) \square \mathrm{cost}(u)$. Hence, we obtain that $\mathrm{cost}\big((s + t) + u\big) = \mathrm{cost}\big(s + (t + u)\big)$.

Case of (A'3). Assume that the transition path starting from s is

$$s \to^{a_{i,1}} s_{i,1} \to^{a_{i,2}} s_{i,2} \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \to^{s_{i,n_i-1}} \checkmark (i = 1, \ldots, N)$$

then we have $\mathrm{cost}(s) = \bigcup_{i=1}^{N}\{\sum_{j=1}^{n_i} a_{ij}\}$. The transition path starting from s + s is

$$s + s \to^{a_{i,1}} s_{i,1} \to^{a_{i,2}} s_{i,2} \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \to^{s_{i,n_i-1}} \checkmark (i = 1, \ldots, N)$$

by the third and fifth rule of Definition 3. We therefore have $\mathrm{cost}(s + s) = \bigcup_{i=1}^{N}\{\sum_{j=1}^{n_i} a_{ij}\}$=cost(s).

Case of (A'4). Assume that the transition paths starting from s, t and u are similar to (A'2). According to the rules in Definition 3, we have

$$(s + t) \square u \to^{a_{i,1}} s_{i,1} \square u \to^{a_{i,2}} s_{i,2} \square u \to \cdots \to^{a_{i,n_i-1}} s_{i,n_i-1} \square u \to^{s_{i,n_i-1}} u$$
$$\to^{c_{j,1}} u_{j,1} \to^{c_{j,2}} u_{j,2} \to \cdots \to^{c_{j,l_j-1}} u_{j,l_j-1} \to^{u_{j,l_j-1}} \checkmark (i = 1, \ldots, N, j = 1, \ldots, L),$$

$$(s + t) \square u \to^{b_{i,1}} t_{i,1} \to^{b_{i,2}} t_{i,2} \to \cdots \to^{b_{i,m_i-1}} t_{i,m_i-1} \to^{t_{i,m_i-1}} u$$

$$\rightarrow^{c_{j,1}} u_{j,1} \rightarrow^{c_{j,2}} u_{j,2} \rightarrow \cdots \rightarrow^{c_{j,l_j-1}} u_{j,l_j-1} \rightarrow^{u_{j,l_j-1}} \checkmark (i = 1, \ldots, M, j = 1, \ldots, L).$$

Hence we know that

$$\mathrm{cost}((s+t)\cdot u) = \bigcup_{i=1}^{N}\bigcup_{j=1}^{L}\left\{\sum_{k=1}^{n_i} a_{ik} + \sum_{k=1}^{l_j} c_{jk}\right\} \cup \bigcup_{i=1}^{M}\bigcup_{j=1}^{L}\left\{\sum_{k=1}^{n_i} b_{ik} + \sum_{k=1}^{l_j} c_{jk}\right\}$$

On the other hand, the transition paths starting from $s \square u$ is

$$s \square u \rightarrow^{a_{i,1}} s_{i,1} \square u \rightarrow^{a_{i,2}} s_{i,2} \square u \rightarrow \cdots \rightarrow^{a_{i,n_i-1}} s_{i,n_i-1} \square u \rightarrow^{s_{i,n_i-1}} u$$

$$\rightarrow^{c_{j,1}} u_{j,1} \rightarrow^{c_{j,2}} u_{j,2} \rightarrow \cdots \rightarrow^{c_{j,l_j-1}} u_{j,l_j-1} \rightarrow^{u_{j,l_j-1}} \checkmark (i = 1, \ldots, N, j = 1, \ldots, L),$$

and the one from $t \square u$ is

$$t \square u \rightarrow^{b_{i,1}} t_{i,1} \rightarrow^{b_{i,2}} t_{i,2} \rightarrow \cdots \rightarrow^{b_{i,m_i-1}} t_{i,m_i-1} \rightarrow^{t_{i,m_i-1}} u$$

$$\rightarrow^{c_{j,1}} u_{j,1} \rightarrow^{c_{j,2}} u_{j,2} \rightarrow \cdots \rightarrow^{c_{j,l_j-1}} u_{j,l_j-1} \rightarrow^{u_{j,l_j-1}} \checkmark (i = 1, \ldots, M, j = 1, \ldots, L).$$

We therefore have $\mathrm{cost}(s \cdot u) = \bigcup_{i=1}^{N}\bigcup_{j=1}^{L}\left\{\sum_{k=1}^{n_i} a_{ik} + \sum_{k=1}^{l_j} c_{jk}\right\}$ and $\mathrm{cost}(t \square u) = \bigcup_{i=1}^{M}\bigcup_{j=1}^{L}\left\{\sum_{k=1}^{n_i} b_{ik} + \sum_{k=1}^{l_j} c_{jk}\right\}$. Consequently, we obtain that $\mathrm{cost}((s+t) \square u) = \mathrm{cost}(s \square u) \square \mathrm{cost}(t \square u)$.

Case of (A'5) Assume that the transition paths starting from s, t and u are similar to (A'2) and (A'4). By the sixth and seventh rules in Definition 3, we have

$$(s \square t) \square u \rightarrow^{a_{i,1}} (s_{i,1} \cdot t) \cdot u \rightarrow^{a_{i,2}} (s_{i,2} \square t) \square u \rightarrow \cdots \rightarrow^{a_{i,n_i-1}} (s_{i,n_i-1} \square t) \square u \rightarrow^{s_{i,n_i-1}} t \square u$$

$$\rightarrow^{b_{j,1}} t_{j,1} \square u \rightarrow^{b_{j,2}} t_{j,2} \square u \rightarrow \cdots \rightarrow^{b_{j,m_j-1}} t_{j,m_j-1} \square u \rightarrow^{t_{j,m_j-1}} u$$

$$\rightarrow^{c_{k,1}} u_{k,1} \rightarrow^{c_{k,2}} u_{k,2} \rightarrow \cdots \rightarrow^{c_{k,l_k-1}} u_{k,l_k-1} \rightarrow^{u_{k,l_k-1}} \checkmark$$

$$(i = 1, \ldots, N;\ j = 1, \ldots, M;\ k = 1, \ldots, L)$$

On the other hand, the transition paths starting from $s \square (t \square u)$ is

$$s \square (t \square u) \rightarrow^{a_{i,1}} s_{i,1} \square (t \square u) \rightarrow^{a_{i,2}} s_{i,2} \square (t \square u) \rightarrow \cdots \rightarrow^{a_{i,n_i-1}} s_{i,n_i-1} \square (t \square u) \rightarrow^{s_{i,n_i-1}} t \square u$$

$$\rightarrow^{b_{j,1}} t_{j,1} \square u \rightarrow^{b_{j,2}} t_{j,2} \square u \rightarrow \cdots \rightarrow^{b_{j,m_j-1}} t_{j,m_j-1} \square u \rightarrow^{t_{j,m_j-1}} u$$

$$\rightarrow^{c_{k,1}} u_{k,1} \rightarrow^{c_{k,2}} u_{k,2} \rightarrow \cdots \rightarrow^{c_{k,l_k-1}} u_{k,l_k-1} \rightarrow^{u_{k,l_k-1}} \checkmark$$

$$(i = 1, \ldots, N;\ j = 1, \ldots, M;\ k = 1, \ldots, L)$$

Both of terms has the same summand as

$$\mathrm{cost}((s \cdot t) \cdot u) = \bigcup_{i=1}^{N} \bigcup_{j=1}^{M} \bigcup_{k=1}^{L} \left\{ \sum_{r=1}^{n_i} a_{i,r} + \sum_{r=1}^{m_j} c_{j,r} + \sum_{r=1}^{l_k} c_{k,r} \right\}$$

$$= \mathrm{cost}(s \cdot (t \cdot u))$$

$$= \mathrm{cost}(s \cdot (t \cdot u))$$

Q.E.D.

Definition 18.A function $\mathrm{cost}^{-1}(-)$of summands is a term such that

$$\mathrm{cost}^{-1}(S) = \sum_{i=1}^{m} a_{i,1} \square a_{i,2} \square \cdots \square a_{i,n_i}$$

where $S = \{a_{i,1} + a_{i,2} + \cdots + a_{i,n_i} \mid i = 1, \ldots, m\}$.Without loss of generality, we can imposea total ordering on the actions and suppose that $a_{i,1} < a_{i,2} < \cdots < a_{i,n_i}$.

Before showing completeness of the formal cost equality with respect to the cost value equality, we show several properties of the function $\mathrm{cost}^{-1}(-)$.

Lemma19. For a processs,$s \sim \mathrm{cost}^{-1}(\mathrm{cost}(s))$.

Proof. We suppose that the transition paths starting from s are

$$s \xrightarrow{a_{i,1}} s_{i,1} \xrightarrow{a_{i,2}} s_{i,2} \rightarrow \cdots \xrightarrow{a_{i,n_i-1}} s_{i,n_i-1} \xrightarrow{a_{i,n_i}} \checkmark (i = 1, \ldots, m)$$

ands' be $\sum_{i=1}^{m} a_{i,1} \square a_{i,2} \square \cdots \square a_{i,n_i}$ where $a_{i,1} < \cdots < a_{i,n_i}$. Then $s \approx s'$ by the definition of cost value equivalence,and therefore$s' = \mathrm{cost}^{-1}(\mathrm{cost}(s')) = \mathrm{cost}^{-1}(\mathrm{cost}(s))$. By the several times of application of (A'1),…,(A'6), we obtain that $s \square s'$. Hence $s \sim \mathrm{cost}^{-1}(\mathrm{cost}(s))$.

Q.E.D.

Lemma 20. For processes sand t, if $s \approx t$ then $\mathrm{cost}^{-1}(\mathrm{cost}(s)) \sim \mathrm{cost}^{-1}(\mathrm{cost}(t))$.

Proof. Suppose that $s \approx t$. By the definition of cost value equality, $\mathrm{cost}(s) = \mathrm{cost}(t)$, and therefore, $\mathrm{cost}^{-1}(\mathrm{cost}(s)) = \mathrm{cost}^{-1}(\mathrm{cost}(t))$.

Q.E.D.

Theorem21. The formal cost equality $s \sim t$ is *complete* with respect to $s \approx t$, in other words, $s \approx t \rightarrow s \sim t$.

Proof.Suppose that $s \approx t$. By Lemma 20, $\mathrm{cost}^{-1}(\mathrm{cost}(s)) \sim \mathrm{cost}^{-1}(\mathrm{cost}(t))$. By Lemma 19, $s \square \mathrm{cost}^{-1}(\mathrm{cost}(s)) \sim \mathrm{cost}^{-1}(\mathrm{cost}(t)) \sim t$.

Q.E.D.

5. EXTENSION TO OTHER SUBSYSTEMS OF ACP

In PAP, the properties that the cost value relation is an equivalence and congruence relation similarly to the case of BPA.

Proposition.22. The cost value equality is an equivalence relation on processes in PAP.

This proposition is proved similarly to the one of BPA. However, the cost value equality is not a congruence relation in PAP. In PAP, the cost value equality satisfies congruence with respect to alternative $+$ and sequential compositions satisfies, but not for merge, left merge $\lfloor\lfloor$, and communication $|$.

Similarly to Lemma 14 and 15, congruence with respect to alternative and sequential composition are proved, respectively.

Proposition 23. For terms s, s', t and t' of PAP, if $s \approx s'$ and $t \approx t'$, then $s + t \approx s' + t'$.

Proposition 23. For terms s, s', t and t' of PAP, if $s \approx s'$ and $t \approx t'$, then $s \; t \approx s' \; t'$.

However, congruence with respect to the other operators does not hold.

Proposition 24. There are terms s, s', t and t' of PAP satisfying that $s \approx s'$ and $t \approx t'$ but $s \; t \not\approx s' \; t'$.

Proof. Let s, s', t and t' be $(a \; b)$, $(c \; d)$, $(b \; a)$ and $(c \; d)$, respectively. Then, the transition paths starting from $(a \; b) \; (c \; d)$ are

$(a \; b) \; (c \; d) \rightarrow^{a} b \; (c \; d) \rightarrow^{b} c \; d \rightarrow^{c} d \rightarrow^{d} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{a} b \; (c \; d) \rightarrow^{c} b \; d \rightarrow^{b} d \rightarrow^{d} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{a} b \; (c \; d) \rightarrow^{c} b \; d \rightarrow^{d} b \rightarrow^{b} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{a} b \; (c \; d) \rightarrow^{c} b \; d \rightarrow^{\gamma(b,d)} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{a} b \; (c \; d) \rightarrow^{\gamma(b,c)} d \rightarrow^{d} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{c} (a \; b) \; d \rightarrow^{a} b \; d \rightarrow^{b} d \rightarrow^{d} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{c} (a \; b) \; d \rightarrow^{a} b \; d \rightarrow^{d} b \rightarrow^{b} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{c} (a \; b) \; d \rightarrow^{a} b \; d \rightarrow^{\gamma(b,d)} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{c} (a \; b) \; d \rightarrow^{d} a \; b \rightarrow^{a} b \rightarrow^{b} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{c} (a \; b) \; d \rightarrow^{\gamma(a,d)} b \rightarrow^{b} \checkmark$,
$(a \; b) \; (c \; d) \rightarrow^{\gamma(a,c)} b \; d \rightarrow^{b} d \rightarrow^{d} \checkmark$
$(a \; b) \; (c \; d) \rightarrow^{\gamma(a,c)} b \; d \rightarrow^{d} b \rightarrow^{b} \checkmark$ and
$(a \; b) \; (c \; d) \rightarrow^{\gamma(a,c)} b \; d \rightarrow^{\gamma(b,d)} \checkmark$.

Hence, $\mathrm{cost}\big((a \cdot b) \; (c \; d)\big) = \{a + b + c + d, a + c + \gamma(b,d), a + b + \gamma(c,d), b + c + \gamma(a,d), b + d + \gamma(a,c), \gamma(a,c) + \gamma(b,d)\}$.

On the other hand the transition paths starting from $(b \; a) \; (c \; d)$ are

$(b \; a) \; (c \; d) \rightarrow^{b} b \; (c \; d) \rightarrow^{a} c \; d \rightarrow^{c} d \rightarrow^{d} \checkmark$,
$(b \; a) \; (c \; d) \rightarrow^{b} b \; (c \; d) \rightarrow^{c} b \; d \rightarrow^{b} d \rightarrow^{d} \checkmark$,
$(b \; a) \; (c \; d) \rightarrow^{b} b \; (c \; d) \rightarrow^{c} b \; d \rightarrow^{d} b \rightarrow^{b} \checkmark$,

$(b \cdot a) \parallel (c \cdot d) \rightarrow^{b} b \cdot (c \cdot d) \rightarrow^{c} b \cdot d \rightarrow^{\gamma(b,d)} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{b} b \cdot (c \cdot d) \rightarrow^{\gamma(a,c)} d \rightarrow^{d} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{c} (b \cdot a) \cdot d \rightarrow^{b} a \cdot d \rightarrow^{a} d \rightarrow^{d} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{c} (b \cdot a) \cdot d \rightarrow^{b} a \cdot d \rightarrow^{d} a \rightarrow^{a} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{c} (b \cdot a) \cdot d \rightarrow^{b} a \cdot d \rightarrow^{\gamma(a,d)} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{c} (b \cdot a) \cdot d \rightarrow^{d} b \cdot a \rightarrow^{b} b \rightarrow^{a} \checkmark$,
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{c} (b \cdot a) \cdot d \rightarrow^{\gamma(b,d)} a \rightarrow^{a} \checkmark$, and
$(b \cdot a) \parallel (c \cdot d) \rightarrow^{\gamma(b,c)} a \cdot d \rightarrow^{\gamma(a,d)} \checkmark$

Hence, $\mathrm{cost}\big((b \cdot a) \parallel (c \cdot d)\big) = \{a + b + c + d, a + c + \gamma(b,d), a + b + \gamma(c,d), b + c + \gamma(a,d), b + d + \gamma(a,c), \gamma(b,c) + \gamma(a,d)\}$.

However, since $\gamma(a,c) + \gamma(b,d) \neq \gamma(b,c) + \gamma(a,d)$, we know $\mathrm{cost}\big((a \cdot b) \parallel (c \cdot d)\big) \neq \mathrm{cost}\big((b \cdot a) \parallel (c \cdot d)\big)$.

Q.E.D.

We have similar properties on left merge $s \lfloor\!\lfloor t$ and communication $s|t$.

The formal cost equality can be extended from BPA to PAP, by adding the rules similar to those in Definition 4. The formal cost equality in BPA also satisfies soundness:

Theorem 25.In PAP, the formal cost equality $s \sim t$ is *sound* with respect to the cost value

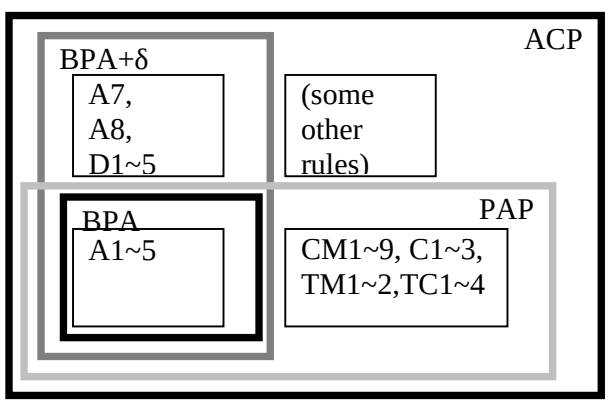

Figure 2.Relationship among sub-fragments of ACP.

equality $s \approx t$, that is, $s \sim t \rightarrow s \approx t$ for terms s and t.

This is proved similarly to the soundnessin BPA (Theorem 17).

The full fragment of ACP is under investigation with respect to cost equivalence. We have another sub-fragment BPA+δ than PAP. ACP is the formal system which is added the mechanism of δ to PAP; on the other hand, BPA+δ is the system which δ is added to BPA.

Definition 26. (Axioms for δ) The axioms for δ is given as follows.

- $x + \delta = x$,, (A7)
- $\delta \cdot x = \delta$, (A8)
- if $v \in H$ then $\partial H(v) = v$, (D1)
- if $v \notin H$ then $\partial H(v) = \delta$, (D2)
- $\partial H(\delta) = \delta$, (D3)
- $\partial H(x + y) = \partial H(x) + \partial H(y)$, (D4)
- $\partial H(x \cdot y) = \partial H(x) \bullet \partial H(y)$, (D5)

The constant δ, called *deadlock* is the action which does not display any behavior. The communication function γ is extended by allowing that the communication of two atomic actions results to δ, that is, $\gamma : A \times A \rightarrow A \cup \{\delta\}$. This extension of γ enables us to express that two actions a and b do not communicate, by defining $\gamma(a, b) = \delta$.

Definition 27 (Transition Relation of δ) The behavior of the encapsulation operators is captured by the following rules

- if $x \xrightarrow{v} \checkmark$ and $v \notin H$, then $\partial H(x) \xrightarrow{v} \checkmark$.
- if $x \xrightarrow{v} x'$ and $v \notin H$ then $\partial H(x) \xrightarrow{v} \partial H(x')$.

Definition 28 (Formal Cost Equivalence for δ) The formal cost equivalence of BPA is extended by adding the constant action δ and the following rules:

- $x + \delta \sim x$, (A'7)
- $\delta \cdot x \sim \delta$, (A'8)
- if $v \in H$ then $\partial H(v) \sim v$, (D'1)
- if $v \notin H$ then $\partial H(v) \sim \delta$, (D'2)
- $\partial H(\delta) \sim \delta$ (D'3)
- $\partial H(x + y) \sim \partial H(x) + \partial H(y)$, (D'4)
- $\partial H(x \cdot y) \sim \partial H(x) \cdot \partial H(y)$, (D'5)

The additional rules for formal cost equivalence are similar to the rules for the axioms for δ. The same cost value function as BPA is introduced to BPA+δ.Then we have the following property.

Proposition 29. The cost value equivalence ≈ in BPA+ δ is an equivalence relation, that is, itsatifies reflexivity, symmetry, and transitivity.

In BPA+ δ, alternative composition satisfies the congruence.

Proposition 30. In BPA+ δ, if $s \approx s'$ and $t \approx t'$ then $s+t \approx s'+t'$.

This is proved similarly to Lemma 14 and Proposition 23.

However, the other operators do not satisfy the congruence.

- There exist processes s, s', t, t' satisfying that $s \approx s'$ and $t \approx t'$ but it does not hold that $s \cdot t \approx s' \cdot t'$.
- There exist processes s and s', satisfying that $s \approx s'$ but it does not hold that $\partial H(s) \approx \partial H(s')$.

4. CONCLUSIONS

We developed a formal system for reasoning about computational costs of concurrent systems in the previous works, based on the Bergstra-Klop's process algebra. We investigated BPA, a subsystem of the Algebra for Communicating Processes extending the mechanism of cost equivalence. We firstly gave the cost value function, secondly defined the formal cost equality, and thirdly studied soundness and completeness for the cost value equivalence and the formal cost equivalence which is obtained by bisimulation of the transition rules.
In future, we should improve the notion of cost value equality and formal cost equality which enjoys congruence and completeness in ACP and the its other subsystems.

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