ON 2-REPEATED SOLID BURST ERRORS

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ABSTRACT

There are several kinds of errors for which error detecting and error correcting codes have been constructed. Solid burst errors are common in many communications. In general communication due to the long messages, the strings of solid bursts of small length may repeat in a vector itself. The concept of repeated bursts is introduced by Beraradi, Dass and Verma[3] which has opened a new area of study. They defined 2-repeated bursts and obtained results for detection and correction of such type of errors.

This paper considers a new similar kind of error which will be termed as '2-repeated solid burst error of length b'. Lower and upper bounds on the number of parity checks required for the existence of codes that detect 2-repeated solid burst error of length b or less are obtained. This is followed by an example of such a code. Further, codes capable of detecting and simultaneously correcting such errors have also been dealt with.

KEYWORDS

Parity check matrix, syndrome, standard array, solid bursts error

1. INTRODUCTION

Investigations in coding theory have been made in several directions but one of the most important aspects considered has been the detection and correction of errors. The nature of errors differs from channel to channel depending upon the behaviour of channels. Due to various channels with the possibility of occurrence of solid burst error (viz. semiconductor memory data [5], supercomputer storage system [1]), solid burst error is one of the important area for mathematicians to study. For study on solid burst, one may refer to [9, 10, 12, etc]. This paper also presents a study on solid burst error, but not occurring in ordinary way.

In very busy communication channels, it is observed that errors repeat themselves. So is a situation when errors need to consider in repeated form. In this direction, an error pattern, called 2-repeated burst has been introduced by Dass, Verma and Berardi [3]. This is an extension of the idea of open-loop burst given by Fire [4]. Later on Dass and Verma [2] defined m-repeated bursts and obtained results regarding the number of parity-check digits required for codes detecting such errors. Some results have been obtained on weights of such errors by Sharma and Rohtagi [11].

This paper introduces yet another kind of a repeated error, termed as '2-repeated solid burst of length b' and presents a study on such errors. The paper is organized as follows. Basic definitions, related to our study are stated with some examples in Section 2. In Section 3, bounds on the parity checks for a code that detects 2-repeated solid bursts of length b or less are obtained. This is followed by an illustration of such a code in section 4. Section 5 gives a bound on code for simultaneous detection and correction of such errors.
In what follows a linear code will be considered as a subspace of the space of all \( n \)-tuples over \( \text{GF}(q) \). The distance between two vectors shall be considered in the Hamming sense.

2. PRELIMINARIES

The definition of a solid burst may be given as follows:

**Definition 1.** A solid burst of length \( b \) is a vector whose all the \( b \)-consecutive components are non-zero and rest are zero.

A vector may have not just one solid burst errors, but more than one. Putting them together into one burst amounts to neglecting the nature of communication and unnecessarily considering a longer solid burst. In a very busy communication channel, sometimes solid bursts repeat themselves. In view of this, a kind of 2-repeated solid burst of length \( b \) is required to be considered. The paper obtains lower and upper bounds for a code detecting 2-repeated solid burst of length \( b \) or less, also presents the case when simultaneous detection and correction is required. A 2-repeated solid burst of length \( b \) may be defined as follows:

**Definition 2.** A 2-repeated solid burst of length \( b \) is a vector of length \( n \) whose only nonzero components are confined consecutively to two distinct sets of \( b \) consecutive components.

As an illustration \((1100011000)\) is a 2-repeated solid burst of length 2 (upto 5) whereas \((0000100100)\) is a 2-repeated burst of length 3 or less. Also the vector \((12221111100)\) may be considered to be a 2-repeated solid burst of length 5 or 6.

The development of codes detecting and correcting such errors may prove to be useful for channels already dealing with multiple solid burst errors improving upon their efficiency as such errors are relatively simpler to handle. Such codes emerge as a natural generalization of single solid burst error detecting and correcting codes.

3. DETECTION OF 2-REPEATED SOLID BURST ERROR

Consider the linear codes that are capable of detecting any 2-repeated solid burst of length \( b \) or less. Clearly, the patterns to be detected should not be code words. In other words, codes are considered which have no 2-repeated solid burst of length \( b \) or less as a code word. Firstly, a lower bound over the number of parity-check digits required for such a code is obtained.

**Theorem 1.** Any \((n, k)\) linear code over \( \text{GF}(q) \) that detects any 2-repeated solid burst of length \( b \) or less must have at least \( 2\log_q \left( \sum_{i=0}^{b} (q-1)^i \right) \) parity-check digits.

**Proof.** The result will be proved on the basis that no detectable error vector can be a code word. Let \( V \) be an \((n, k)\) linear code over \( \text{GF}(q) \). Let \( X \) be the set of all vectors such that all non-zero components are confined consecutively (starting from the positions) in some two distinct fixed set of \( b \) consecutive components.

We claim that no two vectors of the set \( X \) can belong to the same coset of the standard array; else a code word shall be expressible as a sum or difference of two error vectors.

Assume on the contrary that there is a pair, say \( x_1, x_2 \) in \( X \) belonging to the same coset of the standard array. Their difference viz. \( x_1 - x_2 \) must be a code vector. But \( x_1 - x_2 \) is a vector all of
whose non-zero components occur consecutively in the same two fixed \( b \) components and so is a member of \( X \), i.e., \( x_1 \cdot x_2 \) is a 2-repeated solid bursts of length \( b \) or less, which is a contradiction. Thus all the vectors in \( X \) must belong to distinct cosets of the standard array. The number of such vectors, including all zero vector, over \( GF(q) \) is clearly

\[
\left( \sum_{i=0}^{b} (q-1)^i \right)^2. \tag{1}
\]

The theorem follows since there must be at least this number of cosets and number of available is \( q^{n-k} \).

An upper bound on the number of check digits required for the construction of a linear code is provided in the following theorem. This bound assures the existence of a linear code that can detect all 2-repeated solid bursts of length \( b \) or less. The bound has been obtained by the well known technique used in Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [8], also theorem 4.7 Peterson and Weldon [6]).

**Theorem 2.** There exists an \((n, k)\) linear code over \( GF(q) \) that has no 2-repeated solid bursts of length \( b \) or less as a code word provided that

\[
q^{n-k} > 1 + \sum_{i=1}^{b} \sum_{l=0}^{b-1} (q-1)^{isl} (n-i-1). \tag{2}
\]

**Proof.** The existence of such a code will be shown by constructing an appropriate \((n - k) \times n\) parity-check matrix \( H \). The requisite parity-check matrix \( H \) shall be constructed as follows.

Select any non-zero \((n - k)\)-tuples as the first column. Subsequent columns are added to \( H \) such that after having selected the first \( j-1 \) columns \( h_1, h_2, \ldots, h_{j-1} \); the \( j^{th} \) column \( h_j \) is added provided that

\[
h_j \neq (u_i h_{j-1} + u_{i+1} h_{j-2} + \ldots + u_{i+b-1}) + (v_i h_{i+1} + v_{i+1} h_{i+2} + \ldots + v_{i+b-1}) \tag{3}
\]

where \( u_i, v_i \in GF(q); \ s \leq b \) and \( h_i \)'s in the second bracket are any \( s \) consecutive columns among the first \( j-1 \) columns, \( l = 0, 1, 2, \ldots, (b-1) \).

Thus coefficients \( u_i \) form a solid burst of length \( l \) and the coefficients \( v_i \) form a solid burst of length \( b \) or less in a \((j-l-1)\)-tuple. This condition ensures that no 2-repeated solid burst of length \( b \) or less will be a code word. The number of choices of these coefficients can be calculated as follows:

If \( u_i \) is chosen to be a solid burst of length 0, then the number of solid bursts of length \( b \) or less in a \((j-1)\)-tuple is

\[
\sum_{i=0}^{b} (q-1)^i (j-1-i+1). \tag{4}
\]

If \( u_i \) is chosen to be a solid burst of length 1, then the number of solid bursts of length \( b \) or less in a \((j-2)\)-tuple is
\begin{equation}
\sum_{i=1}^{b} (q-1)^i (j - 2 - i + 1). \tag{5}
\end{equation}

If \( u_i \) is chosen to be a solid burst of length 3, then the number of solid bursts of length \( b \) or less in a \((j-3)\)-tuple is
\begin{equation}
\sum_{i=1}^{b} (q-1)^i (j - 3 - i + 1). \tag{6}
\end{equation}

Continuing the process, if \( u_i \) is chosen to be a solid burst of length \( b-1 \), then the number of solid bursts of length \( b \) or less in a \((j-b)\)-tuple is
\begin{equation}
\sum_{i=1}^{b} (q-1)^i (j - b - i + 1). \tag{7}
\end{equation}

Therefore, the total number of possible choices of the coefficients \( u_i \) and \( v_j \), is
\[
\sum_{i=1}^{b} (q-1)^i (j - 1 - i + 1) + (q-1) \sum_{i=1}^{b} (q-1)^i (j - 2 - i + 1) + (q-1)^i \sum_{i=1}^{b} (q-1)^i (j - 3 - i + 1) \\
+ \ldots + (q-1)^{b-1} \sum_{i=1}^{b} (q-1)^i (j - b - i + 1),
\]
which is equal to
\begin{equation}
\sum_{i=1}^{b} \sum_{l=0}^{b-1} (q-1)^{i+l} (j - i - l). \tag{8}
\end{equation}

At worst, all these linear combinations might yield a distinct sum.

Therefore a column \( h_j \) can be added to \( H \) provided that
\begin{equation}
q^{n-k} > 1 + \sum_{i=1}^{b} \sum_{l=0}^{b-1} (q-1)^{i+l} (j - i - l). \tag{9}
\end{equation}

For a code of length \( n \), replacing \( j \) by \( n \) gives the result. \( \square \)

4. ILLUSTRATION

Example. Consider a \((7, 2)\) binary code with parity check matrix
\[
H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}
\]
This matrix has been constructed by the synthesis procedure, outlined in the proof of Theorem 2, by taking $q = 2$, $n = 7$ and $b = 2$. It can be seen from Table 1 that the syndromes of the different 2-repeated solid bursts of length 2 or less are nonzero, showing thereby that the code that is the null space of this matrix can detect all 2-repeated solid bursts of length 2 or less.

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5. SIMULTANEOUS DETECTION AND CORRECTION OF 2-REPEATED SOLID BURST ERRORS

This section determines extended Reiger's bound (refer [8]; also Theorem 4.15, Peterson and Weldon[7] ) for simultaneous detection and correction of 2-repeated solid bursts of length $b$ or less. The following theorem gives a bound on the number of parity-check digits for a linear code that simultaneously detects and corrects such errors.

**Theorem 3.** An $(n, k)$ linear code over GF($q$) that corrects all 2- repeated solid bursts of length $b$ or less must have at least $2log_q \left\{ \sum_{i=0}^{2b} (q - 1)^i \right\}$ parity-check digits. Further, if the code corrects all
2-repeated solid bursts of length \(b\) or less and simultaneously detects 2-repeated solid bursts of length \(d\) \((d > b)\) or less then the code must have at least \(2\log_q \left\{ \sum_{i=0}^{b+d} (q-1)^i \right\}\) parity-check digits.

**Proof.** To prove the first part, consider a solid burst of length \(4b\) or less. Such a vector is expressible as a sum or difference of two vectors, each of which is a 2-repeated solid burst of length \(b\) or less. These component vectors must belong to different cosets of the standard array because both such errors are correctable errors. Accordingly, such a vector viz. solid burst of length \(4b\) or less cannot be a code vector. A solid burst of length \(4b\) or less is also a 2-repeated solid burst of length \(2b\) or less. Applying Theorem 1, such a code must have at least 

\[2\log_q \left\{ \sum_{i=0}^{b} (q-1)^i \right\}\] 

parity-check digits.

Further, consider a solid burst of length \(2(b + d)\) or less. Such a vector is expressible as a sum or difference of two vectors, one of which is a 2-repeated solid burst of length \(b\) or less and the other is a 2-repeated solid burst of length \(d\) or less. Both such component vectors, one being a detectable error and the other being a correctable error, cannot belong to the same coset of the standard array. Therefore such a vector cannot be a code vector, i.e., a 2-repeated solid burst of length \(b + d\) or less cannot be a code vector. Hence the code must have at least 

\[2\log_q \left\{ \sum_{i=0}^{b+d} (q-1)^i \right\}\] 

parity check digits.

6. CONCLUSION

The paper presents lower and upper bounds for a code detecting 2-repeated solid burst of length \(b\) or less, also presents the case when simultaneous detection and correction is required. As correction of errors, if possible, is more important than detection of errors, the codes correcting such errors may be quite useful study.

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