COMPARATIVE ANALYSIS OF AHP AND FUZZY AHP MODELS FOR MULTICRITERIA INVENTORY CLASSIFICATION

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ABSTRACT

A systematic approach to the inventory control and classification may have a significant influence on company competitiveness. In practice, all inventories cannot be controlled with equal attention. In order to efficiently control the inventory items and to determine the suitable ordering policies for them, multi-criteria inventory classification is used. Analytical Hierarchy Process (AHP) is one of the best ways for deciding among the complex criteria structure in different levels. Fuzzy Analytical Hierarchy Process (FAHP) is a synthetic extension of classical AHP method when the fuzziness of the decision makers is considered. In this paper, a comparative analysis of AHP and FAHP for multi-criteria inventory classification model has been presented. To accredit the proposed models, those were implemented for the 351 raw materials of switch gear section of Energypac Engineering Limited (EEL), a large power engineering company of Bangladesh.

KEYWORDS

Analytic Hierarchy Process, Chang’s Extent Analysis, Inventory Classification

1. INTRODUCTION

Inventory has been looked at as a major cost and source of uncertainty due to the volatility within the commodity market and demand for the value-added product. Inventory is held by manufacturing companies for a number of reasons, such as to allow for flexible production schedules and to take advantage of economies of scale when ordering stock. The efficient management of inventory systems is therefore a crucial element in the operation of any production or manufacturing company. Classification of inventory is a crucial element in the operation of any production company. Because of the huge number of inventory items in many companies, great attention is directed to inventory classification into the different classes, which consequently require the application of different management tools and policies (Chase et al., 2006). ABC inventory management deals with classification of the items in an inventory in decreasing order of annual dollar volume. The ABC classification process is an analysis of a range of items, such as finished products or customers into three categories: A - outstandingly important; B - of average importance; C - relatively unimportant as a basis for a control scheme. Each category can and sometimes should be handled in a different way, with more attention being devoted to category A, less to B, and less to C (Kabir & Hasin, 2011a).
Sometimes, only one criterion is not a very efficient measure for decision-making. Therefore, multiple criteria decision making methods are used (Flores & Whybark, 1986, 1987). Apart from other criteria like lead time of supply, part criticality, availability, stock out penalty costs, ordering cost, scarcity, durability, substitutability, reparability etc has been taken into consideration (Flores & Whybark, 1986, 1987; Zhou & Fan, 2007). More studies have been done on multi-criteria inventory classification in the past 20 years. So many different methods for classifying inventory and taking into consideration multiple criteria have been used and developed.

Flores & Whybark (1986, 1987) proposed the bi-criteria matrix approach, wherein annual dollar usage by a joint criteria matrix is combined with another criterion. Flores et al. (1992) have proposed the use of joint criteria matrix for two criteria. Analytic Hierarchy Process (AHP) developed by Saaty (1980) has been successfully applied to Multicriteria inventory classification by Flores et al. (1992). The advantage of the AHP is that it can incorporate many criteria and ease of use on a massive accounting and measurement system, but its shortcoming is that a significant amount of subjectivity is involved in pairwise comparisons of criteria. They have used the AHP to reduce multiple criteria to a univariate and consistent measure. However, Flores et al. (1992) has taken average unit cost and annual dollar usage as two different criteria among others. The problem with this approach is that the annual dollar usage and the unit price of items are usually measured in different units.

Partovi & Burton (1993) applied the analytic hierarchy process (AHP) to inventory classification in order to include both quantitative and qualitative evaluation criteria. Braglia et al., (2004) integrated decision diagram with a set of analytic hierarchy process (AHP) models used to solve the various multi-attribute decision sub-problems at the different levels/nodes of the decision tree. Guvenir & Erel (1998) applied genetic algorithm technique to the problem of multi-criteria inventory classification. Their proposed method is called Genetic Algorithm for Multicriteria Inventory Classification and it uses genetic algorithm to learn the weights of criteria. Partovi & Anandarajan (2002) proposed an artificial neural network (ANN) approach for inventory classification. Lei et al. (2005) compared principal component analysis with a hybrid model combining principal component analysis with artificial neural network and back propagation algorithm. Ramanathan (2006) proposed a weighted linear optimization model for multiple criteria ABC inventory classification, where performance score of each item obtained using a Data Envelopment Analysis (DEA)-like model. Liu & Huang (2006) present a modified Data Envelopment Analysis (DEA) model to address ABC inventory classification.

Bhattacharya et al. (2007) developed a distance-based multiple-criteria consensus framework utilizing the technique for order preference by similarity to ideal solution (TOPSIS) for ABC analysis. Ng (2007) proposes a weighted linear model for MCABC inventory classification. Via a proper transformation, the Ng model can obtain the scores of inventory items without a linear optimizer. Chen et al. (2008) proposed a case-based distance model for multiple criteria ABC analysis. Jamshidi & Jain (2008) addressed multi-criteria ABC inventory classification to standardized each criterion and weight them for classification. Šimunović et al. (2009) investigated the application of neural networks in multiple criteria inventory classification. Hadi-Vencheh (2010) proposed a simple nonlinear programming model which determines a common set of weights for all the items. Yu (2011) compared artificial-intelligence (AI)-based classification techniques with traditional multiple discriminant analysis (MDA).

Therefore the main objective of this research is to present a comparative analysis Analytic Hierarchy Process (AHP) and Fuzzy Analytic Hierarchy Process (FAHP) for multi-criteria inventory classifications. In the flow of the paper, first the classical AHP and Fuzzy AHP methods are introduced, then the summary of calculations are presented as the next section. Finally, the paper ends with comparison results, findings, and comments about these methods.
2. **Analytic Hierarchy Process (AHP)**

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach and was introduced by Saaty (1980). AHP organizes the basic rationality by breaking down a problem into its smaller constituent parts. By decomposing the problem, the decision-maker can focus on a limited number of items at the same time. The AHP is carried out in two phases: the design of the hierarchy and the evaluation of the components in the hierarchy (Saaty, 1980; Vargas, 1990).

AHP is a multi criteria decision making process that is especially suitable for complex decisions which involve the comparison of decision elements which are difficult to quantify. It is based on the assumption that when faced with a complex decision the natural human reaction is to cluster the decision elements according to their common characteristics. It is a technique for decision making where there are a limited number of choices, but where each has a number of different attributes, some or all of which may be difficult to formalize. It is especially applicable when a team is making decisions. It involves building a hierarchy (Ranking) of decision elements and then making comparisons between each possible pair in each cluster (as a matrix). This gives a weighting for each element within a cluster (or level of the hierarchy) and a consistency ratio (useful for checking the consistency of the data).

The crux of AHP is the determination of the relative weights to rank the decision alternatives. Assuming that there are \( n \) criteria at a given hierarchy, the procedure establishes an \( n \times n \) pair-wise comparison matrix, \( A \), that reflects the decision maker’s judgment of the relative importance of the different criteria. The pair-wise comparison is made such that the criterion in row \( i \) (\( i = 1,2,3,\ldots,n \)) is ranked relative to each of the criteria represented by the \( n \) columns. Letting \( a_{ij} \) define the element \((i,j)\) of \( A \), AHP uses a discrete scale from 1 to 9 in which \( a_{ij} = 1 \) signifies that \( i \) and \( j \) are equally important, \( a_{ij} = 5 \) indicates that \( i \) is strongly more important than \( j \) and \( a_{ij} = 9 \) indicates that \( i \) is extremely more important than \( j \). Other intermediate values between 1 and 9 are interpreted correspondingly. For consistency, \( a_{ij} \cdot k \) automatically implies that \( a_{ji} = 1/k \). Also all the diagonal elements \( a_{ii} \) of \( A \) must equal 1 because they rank a criterion against itself. The relative weights of criterion can be determined from \( A \) by dividing the elements of each column by the sum of the elements of the same column. The resulting matrix is called normalized matrix, \( N \).

The numerical results of attributes are presented to the decision maker to assign relative importance according to a predefined scale. Now a judgment matrix prepared. It is an \((n \times n)\) matrix; normalized weights are calculated as follows.

\[
\begin{array}{cccc}
1 & 2 & \cdots & n \\
1 & a_{12} & \cdots & a_{1n} \\
a_{21} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{k1} & a_{k2} & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn} \\
\end{array}
\]

Where, \( i \) and \( j \) are the alternatives to be compared. \( a_{ij} \) is a value that represent comparison between alternatives or attributes \( i \) and \( j \).
The above judgment matrix may be consistent if $a_{ij} \cdot a_{jk} = a_{ik}$

For all values of $i, j, k$, in the above judgment matrix, sum of the element in a column, $y_k = \sum a_{ij}$, where, $i = 1, 2, \ldots n$ and $j = 1, 2, \ldots n$

Geometric mean is calculated as, $b_k = [(a_{k1}) \cdot (a_{k2}) \cdot \ldots (a_{kn})]^{1/n}$, where, $k = 1, 2, \ldots n$

Normalized weights are calculated as, $X_k = \frac{b_k}{\sum b_k}$

Acceptability of alternative or attribute is measured in terms of Consistency Ratio (C.R.)

$$
\text{Consistency ratio} = \frac{\text{consistency index}}{\text{randomly generated consistency index}}
$$

Atty’s measure of consistency is done in terms of Consistency Index (C.I.) $\text{C.I.} = \frac{\lambda_{\text{max}} - n}{n - 1}$

where, $\lambda_{\text{max}} = y_1x_1 + y_2x_2 + \ldots + y_kx_k + \ldots + y_nx_n = \sum y_kx_k$ = largest eigen value of matrix of order $n$

Now, some Randomly Generated Consistency Index (R.I.) values are given in Table 1.

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. I.</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

The acceptable CR range varies according to the size of matrix i.e. 0.05 for a 3 by 3 matrix, 0.08 for a 4 by 4 matrix and 0.1 for all larger matrices, $n \geq 5$. If the value of CR is equal to, or less than that value, it implies that the evaluation within the matrix is acceptable or indicates a good level of consistency in the comparative judgments represented in that matrix. In contrast, if CR is more than the acceptable value, inconsistency of judgments within that matrix has occurred and the evaluation process should therefore be reviewed, reconsidered and improved. An acceptable consistency ratio helps to ensure decision-maker reliability in determining the priorities of a set of criteria (Saaty, 2000, Kabir & Sumi, 2010).

### 3. Why FAHP in stead of AHP?

In the conventional AHP, the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale. So, the application of Saaty's AHP has some shortcomings as follows (Kabir & Hasin, 2011b): (1) The AHP method is mainly used in nearly crisp decision applications, (2) The AHP method creates and deals with a very unbalanced scale of judgment, (3) The AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number, (4) Ranking of the AHP method is rather imprecise, (5) The subjective judgment, selection and preference of decision-makers have great influence on the AHP results. In addition, a decision-maker's requirements on evaluating alternatives always contain ambiguity and multiplicity of meaning. Furthermore, it is also recognized that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore, conventional AHP seems inadequate to capture decision maker's requirements explicitly (Kabir & Hasin, 2011b).

In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pairwise comparison as an extension of AHP. A variant of AHP, called Fuzzy AHP, comes into implementation in order to overcome the compensatory approach and the inability of the AHP in handling linguistic variables. The fuzzy AHP approach allows a more accurate description of the decision making process.
4. FUZZY ANALYTIC HIERARCHY PROCESS

The fuzzy AHP technique can be viewed as an advanced analytical method developed from the traditional AHP. Generally, it is impossible to reflect the decision makers’ uncertain preferences through crisp values. Therefore, FAHP is proposed to relieve the uncertainty of AHP method, where the fuzzy comparisons ratios are used. There are the several procedures to attain the priorities in FAHP. The fuzzy least square method (Xu, 2000), method based on the fuzzy modification of the LLSM (Boender et al., 1989), geometric mean method (Buckley, 1985), the direct fuzzification of the method of Csutora and Buckley (2001), synthetic extend analysis (Chang, 1996), Mikhailov’s fuzzy preference programming (Mikhailov, 2003) and two-stage logarithmic programming (Wang et al., 2005) are some of these methods. Chang’s extent analysis is utilized in this research to evaluate the focusing problem.

Chang (1992) introduces a new approach for handling pair-wise comparison scale based on triangular fuzzy numbers followed by use of extent analysis method for synthetic extent value of the pairwise comparison (Chang, 1996). The first step in this method is to use triangular fuzzy numbers for pairwise comparison by means of FAHP scale, and the next step is to use extent analysis method to obtain priority weights by using synthetic extent values. The fuzzy evaluation matrix of the criteria was constructed through the pairwise comparison of different attributes relevant to the overall objective using the linguistic variables and triangular fuzzy numbers (Figure 1 and Table 2).

![Figure 1. Linguistic Variables for the Importance Weight of Each Criterion](image)

<table>
<thead>
<tr>
<th>Linguistic scale for importance</th>
<th>Fuzzy numbers</th>
<th>Membership function</th>
<th>Domain</th>
<th>Triangular fuzzy scale ((l, m, u))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Just equal</td>
<td>1</td>
<td>(\mu_M(x) = \frac{(3-x)}{(3-1)}) (1 \leq x \leq 3)</td>
<td>((1, 1, 1))</td>
<td></td>
</tr>
<tr>
<td>Equally important</td>
<td>3</td>
<td>(\mu_M(x) = \frac{(x-1)}{(3-1)}) (1 \leq x \leq 3)</td>
<td>((1, 1, 3))</td>
<td></td>
</tr>
<tr>
<td>Weakly important</td>
<td>5</td>
<td>(\mu_M(x) = \frac{(5-x)}{(5-3)}) (3 \leq x \leq 5)</td>
<td>((1, 3, 5))</td>
<td></td>
</tr>
<tr>
<td>Essential or Strongly important</td>
<td>7</td>
<td>(\mu_M(x) = \frac{(7-x)}{(7-5)}) (5 \leq x \leq 7)</td>
<td>((3, 5, 7))</td>
<td></td>
</tr>
<tr>
<td>Very strongly important</td>
<td>9</td>
<td>(\mu_M(x) = \frac{(9-x)}{(9-7)}) (7 \leq x \leq 9)</td>
<td>((5, 7, 9))</td>
<td></td>
</tr>
<tr>
<td>Extremely preferred</td>
<td>11</td>
<td>(\mu_M(x) = \frac{(x-7)}{(9-7)}) (9 \leq x \leq 11)</td>
<td>((7, 9, 9))</td>
<td></td>
</tr>
</tbody>
</table>

If factor \(i\) has one of the above numbers assigned to it when compared to factor \(j\), then \(j\) has the reciprocal value when compared to \(i\). Reciprocals of above \(M_1^{-1} = \frac{1}{u_1 \cdot m_1 \cdot l_1} \)
Source: Kabir & Hasin (2011a)

The following section outlines the Chang’s extent analysis method on FAHP. Let \( X = \{x_1, x_2, \ldots, x_n\} \) be an object set and \( U = \{u_1, u_2, \ldots, u_m\} \) be a goal set. As per Chang (1992, 1996) each object is taken and analysis for each goal, \( g_i \), is performed, respectively. Therefore \( m \) extent analysis values for each object can be obtained, as under:

\[
M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m, \quad i = 1, 2, 3, \ldots, n
\]

where all the \( M_{g_i}^j \) (\( j = 1, 2, \ldots, m \)) are TFNs whose parameters are, depicting least, most and largest possible values respectively and represented as \((a, b, c)\).

The steps of Chang’s extent analysis (Chang, 1992) can be detailed as follows (Bozbura et al., 2007; Kahraman et al., 2004; Kabir & Hasin, 2011b):

**Step 1:** The value of fuzzy synthetic extent with respect to \( i \) th object is defined as

\[
S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[ \sum_{j=1}^{m} M_{g_i}^j \right]^{-1}
\]

To obtain \( \sum_{j=1}^{m} M_{g_i}^j \), perform the fuzzy addition operation of \( m \) extent analysis values for a particular matrix such that

\[
\sum_{j=1}^{m} M_{g_i}^j = (\sum_{j=1}^{m} a_j, \sum_{j=1}^{m} b_j, \sum_{j=1}^{m} c_j)
\]

And to obtain \( \sum_{j=1}^{m} \sum_{i=1}^{n} M_{g_i}^j \) perform the fuzzy addition operation of \( M_{g_i}^m \) \( (j = 1, 2, \ldots, m) \) values such that

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j = (\sum_{i=1}^{n} a_i, \sum_{i=1}^{n} b_i, \sum_{i=1}^{n} c_i)
\]

And then compute the inverse of the vector in equation (11) such that

\[
[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j]^{-1} = \left( \frac{1}{\sum_{i=1}^{n} a_i}, \frac{1}{\sum_{i=1}^{n} b_i}, \frac{1}{\sum_{i=1}^{n} c_i} \right)
\]

**Step 2:** The degree of possibility of \( M_2 = (a_2, b_2, c_2) \) \( \geq M_1 = (a_1, b_1, c_1) \) is defined as

\[
V(M_2 \geq M_1) = \sup \{\min (\mu_{M_1}(x), \mu_{M_2}(x))\}
\]

And can be equivalently expressed as follows:

\[
V(M_2 \geq M_1) = \begin{cases} 
1, & \text{if } b_2 \geq b_1 \\
0, & \text{if } a_1 \geq c_2 \\
\frac{a_1 - c_2}{(b_2 - c_2) - (b_1 - a_1)}, & \text{otherwise}
\end{cases}
\]

where \( d \) is the ordinate of the highest intersection point \( D \) between \( \mu_{M_1} \) and \( \mu_{M_2} \) as shown in Figure 2.
To compare $M_1$ and $M_2$, both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

**Step 3:** The degree of possibility for a convex fuzzy number to be greater than $k$ convex fuzzy numbers $M_i \ (i = 1,2,\ldots,k)$ can be defined by

$$V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1) \land (M \geq M_2) \land \ldots (M \geq M_k)]$$

$$= \min V(M \geq M_i), \ (i = 1, 2, 3, \ldots, k)$$

Assuming that

$$d'(A_i) = \min V(S_i \geq S_k)$$

for $k = 1, 2, 3, \ldots, n; \ k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$$

where $A_i = (i = 1, 2, 3, \ldots, n)$ are $n$ elements

**Step 4:** By normalizing, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T$$

where $W$ is a non-fuzzy number.

5. **APPLICATION OF THE MODELS**

To accredit the proposed model, it is implemented for the 351 raw materials of switch gear section of Energypac Engineering Limited (EEL), one of the leading power engineering companies in Bangladesh. Energypac Engineering Ltd. is the manufacturer of Transformer (Power Transformer, Distribution Transformer and Instrumental Transformer) and Switchgear (Outdoor vacuum circuit breaker, Indoor vacuum circuit breaker, Control, Metering and Relay panels, Low Tension and Power Factor Improvement panel, Indoor type Load Break Switch, Outdoor Offload disconnector and By-pass switch). Analytic Hierarchy Process (AHP) and Fuzzy Analytic Hierarchy Process (Fuzzy AHP) are used to determine the relative weights of the attributes or criterions and to classify inventories into different categories through training the data set.

5.1 **Determination of Criteria**

Based on the extensive literature review given in the previous section, experts participating in the implementation of this model have regarded five important criteria for classification of inventory. Those are: Unit Price, Annual Demand, Criticality, Last Use Date and Durability.

5.2 **Pairwise Comparison of Criteria**

For Multicriteria inventory classification, a questionnaire was designed to elicit judgments about
the relative importance of each of the selected criteria for AHP and Fuzzy AHP models (Appendix A). The questionnaire was completed by fourteen experts, among them three academia’s and eleven professional including raw material and inventory manager of EEL. To aggregate group decisions, Geometric Mean operations are used for both AHP and Fuzzy AHP models.

5.3 Data Collection

Unit price, last year consumption or annual demand, last use date, criticality, durability of 351 materials of switch gear section has been collected. Range and value for the transformation of last use date, criticality and durability are shown in Table 3-5.

Table 3. Transformation of Last Use Date

<table>
<thead>
<tr>
<th>Range</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used within a day</td>
<td>10</td>
</tr>
<tr>
<td>Used within a week</td>
<td>8</td>
</tr>
<tr>
<td>Used within a month</td>
<td>6</td>
</tr>
<tr>
<td>Used within 6 month</td>
<td>4</td>
</tr>
<tr>
<td>Used within a year</td>
<td>2</td>
</tr>
<tr>
<td>Used more than a year</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4. Transformation of Criticality

<table>
<thead>
<tr>
<th>Range</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely Critical</td>
<td>5</td>
</tr>
<tr>
<td>Moderate Critical</td>
<td>3</td>
</tr>
<tr>
<td>Non Critical</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5. Transformation of Durability

<table>
<thead>
<tr>
<th>Mean Time Between Failure</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 1 Week</td>
<td>10</td>
</tr>
<tr>
<td>&gt; 1 Month</td>
<td>8</td>
</tr>
<tr>
<td>&gt; 6 Month</td>
<td>5</td>
</tr>
<tr>
<td>&gt; 1 year</td>
<td>3</td>
</tr>
<tr>
<td>&lt; 1 year</td>
<td>1</td>
</tr>
</tbody>
</table>

5.4 Determination of the Weights of Criteria Using AHP

The aggregated pairwise comparison matrix of the fourteen experts or decision maker’s with respect to the overall objective is shown in Table 6.

Table 6. Aggregated Comparison Matrix of the Attributes for AHP Model

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Unit Price</th>
<th>Annual Demand</th>
<th>Criticality</th>
<th>Last Use Date</th>
<th>Durability</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Price</td>
<td>1</td>
<td>1.602</td>
<td>1.076</td>
<td>1.626</td>
<td>1.843</td>
<td>0.272</td>
</tr>
<tr>
<td>Annual Demand</td>
<td>0.625</td>
<td>1</td>
<td>3.294</td>
<td>1.104</td>
<td>2.784</td>
<td>0.283</td>
</tr>
<tr>
<td>Criticality</td>
<td>0.93</td>
<td>0.242</td>
<td>1</td>
<td>1.116</td>
<td>1.17</td>
<td>0.153</td>
</tr>
<tr>
<td>Last Use Date</td>
<td>0.615</td>
<td>0.72</td>
<td>0.713</td>
<td>1</td>
<td>1.06</td>
<td>0.157</td>
</tr>
<tr>
<td>Durability</td>
<td>0.543</td>
<td>0.359</td>
<td>0.855</td>
<td>0.944</td>
<td>1</td>
<td>0.135</td>
</tr>
</tbody>
</table>

The maximum value of eigen vector for the above matrix, \( \lambda_{\text{max}} = 5.1527 \)
Consistency index, \( C.I. = (\lambda_{\text{max}} - n)/(n-1) = 0.03816 \)

Random Index for the matrix of order 5, \( R.I. = 1.12 \)

Consistency Ratio, \( C.R. = C.I./R.I. = 0.03407 \). As CR < 0.1 the level of inconsistency present in the information stored in comparison matrix is satisfactory (Saaty, 1998).

The weight of each success factor is depicted in Figure 3. Figure 3 show that the annual demand has higher priority than the other criteria. The weights of the criteria represent the ratio of how much more important one criterion is than another, with respect to the goal or criterion at a higher level.

\[
\begin{array}{cccccc}
\text{Unit Price} & \text{Annual Demand} & \text{Criticality} & \text{Last Use Date} & \text{Durability} & \text{Weights} \\
0.00 & 0.05 & 0.10 & 0.15 & 0.20 & 0.25 & 0.30 \\
\end{array}
\]

![Figure 3. Weights of Criteria Using AHP Model](image)

### 5.5 Determination of Composite Priority Weights Using AHP

In AHP methodology, for a very large number of alternatives (351), making pair wise comparisons of alternatives, with respect to each criterion, can be time consuming and confusing, because the total number of comparisons will also be very big. Therefore, multiple criteria inventory classification is carried out by using the modified AHP methodology, which includes pair wise comparisons of criteria, but not pair wise comparisons of alternatives. Because of the large number of alternatives (351), pair wise comparisons of the alternatives are not performed.

Finally, the composite priority weights of each alternative can be calculated by multiplying the weights of each alternative by the data of the corresponding criteria. The composite priority weight of the alternatives gives the idea about the appropriate class of the alternatives or items. Items are ranked according to overall composite priority weights in the descending order. The limits for the classes are derived on the following basis. Class A involves 70% of the total composite priority weights. Class B involves 20% of the total composite priority weights amount of items, while 10% of total composite priority weights belong to class C. The results of the study show that among 351 items 22 items are identified as class A or very important group or outstandingly important, 45 items as class B or important group or average important and the remaining 284 items as class C or unimportant group or relatively unimportant as a basis for a control scheme using AHP model.

### 5.6 Determination of the Weights of Criteria Using FAHP

As mentioned before, these classical AHP results are compared with the fuzzy AHP results. Therefore, the evaluations are recalculated according to the fuzzy AHP on the same raw materials for inventory classification. Table 7 shows the aggregated fuzzy pairwise comparisons of the fourteen experts or decision maker's. The aggregated decision matrix as shown in Table 3 is
constructed to measure the relative degree of importance for each criterion, based on the Chang’s extent analysis.

Table 7. Aggregated Fuzzy Comparison Matrix of the Attributes for Fuzzy AHP Model

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Unit Price</th>
<th>Annual Demand</th>
<th>Criticality</th>
<th>Last Use Date</th>
<th>Durability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Price</td>
<td>1,1,1</td>
<td>0.89,1.62,2.25</td>
<td>0.65,1.07,1.88</td>
<td>0.82,1.47,2.76</td>
<td>0.8,1.37,3.19</td>
</tr>
<tr>
<td>Annual Demand</td>
<td>0.44,0.62,1.12</td>
<td>1,1</td>
<td>2.02,3.08,4.64</td>
<td>0.80,1.14,1.72</td>
<td>1.17,2.36,4.53</td>
</tr>
<tr>
<td>Criticality</td>
<td>0.53,0.93,1.53</td>
<td>0.22,0.34,0.50</td>
<td>1,1</td>
<td>0.68,1.11,1.66</td>
<td>0.80,1.17,2.12</td>
</tr>
<tr>
<td>Last Use Date</td>
<td>0.36,0.68,1.21</td>
<td>0.68,1.12,1.26</td>
<td>0.60,0.90,1.47</td>
<td>1,1</td>
<td>0.76,0.93,1.25</td>
</tr>
<tr>
<td>Durability</td>
<td>0.31,0.73,1.26</td>
<td>0.22,0.42,0.86</td>
<td>0.58,1.12,1.26</td>
<td>0.80,1.08,1.32</td>
<td>1,1</td>
</tr>
</tbody>
</table>

Inconsistency of TFN used can be checked and the consistency ratio (CR) has to calculate. The results obtained are: largest eigen value of matrix, \( \lambda_{\text{max}} = 5.323 \); Consistency Index (C.I.) = 0.08075; Randomly Generated Consistency Index (R.I.) = 1.12 and Consistency Ratio (C.R.) = 0.0721 As CR < 0.1 the level of inconsistency present in the information stored in comparison matrix is satisfactory.

\[ S_U = (4.16, 6.51, 11.08) \otimes (1/4.16, 1/27.68, 1/19.13) = (0.09, 0.235, 0.58) \]
\[ S_A = (5.43, 8.06, 12.76) \otimes (1/4.16, 1/27.68, 1/19.13) = (0.13, 0.291, 0.67) \]
\[ S_C = (3.23, 4.38, 6.41) \otimes (1/4.16, 1/27.68, 1/19.13) = (0.077, 0.158, 0.34) \]
\[ S_L = (3.4, 4.51, 6.19) \otimes (1/4.16, 1/27.68, 1/19.13) = (0.08, 0.163, 0.32) \]
\[ S_D = (2.91, 4.23, 5.7) \otimes (1/4.16, 1/27.68, 1/19.13) = (0.07, 0.153, 0.30) \]

The degree of possibility of superiority of \( S_U \) is calculated and is denoted by \( V (S_U \geq S_A) \). Therefore, the degree of possibility of superiority for the first requirement- the values are calculated as

\[ V (S_U \geq S_A) = 0.9, \quad V (S_U \geq S_C) = 1, \quad V (S_U \geq S_L) = 1, \quad V (S_U \geq S_D) = 1, \]

For the second requirement- the values are calculated as

\[ V (S_A \geq S_U) = 1, \quad V (S_A \geq S_C) = 1, \quad V (S_A \geq S_L) = 1, \quad V (S_A \geq S_D) = 1, \]

For the third requirement- the values are calculated as

\[ V (S_C \geq S_U) = 0.75, \quad V (S_C \geq S_A) = 0.61, \quad V (S_C \geq S_L) = 1, \quad V (S_C \geq S_D) = 1, \]

For the fourth requirement- the values are calculated as

\[ V (S_L \geq S_U) = 0.75, \quad V (S_L \geq S_A) = 0.60, \quad V (S_L \geq S_C) = 1, \quad V (S_L \geq S_D) = 1, \]

For the fifth requirement- the values are calculated as

\[ V (S_D \geq S_U) = 0.70, \quad V (S_D \geq S_A) = 0.55, \quad V (S_D \geq S_C) = 0.98, \quad V (S_D \geq S_L) = 0.96, \]

The minimum degree of possibility of superiority of each criterion over another is obtained. This further decides the weight vectors of the criteria. Therefore, the weight vector is given as
$W' = (0.9, 1, 0.61, 0.60, 0.55)$

The normalized value of this vector decides the priority weights of each criterion over another. The normalized weight vectors are calculated as

$W = (0.246, 0.273, 0.167, 0.164, 0.15)$

The normalized weight of each success factor is depicted in Figure 4. Figure 4 show that still annual demand has higher priority than the other criteria.

![Weights of Criteria Using Fuzzy AHP Model](image)

**Figure 4:** Weights of Criteria Using Fuzzy AHP Model

### 5.7 Determination of Composite Priority Weights Using Fuzzy AHP

Finally, the composite priority weights of each alternative also calculated using modified FAHP methodology. Class A involves 70 %, Class B involves 20 % and Class C involves remaining 10 % of the total composite priority weights. The results of the study show that among 351 items 22 items are identified as class A or very important group or outstandingly important, 47 items as class B or important group or average important and the remaining 282 items as class C or unimportant group or relatively unimportant as a basis for a control scheme using Fuzzy AHP model.

### 6. DISCUSSIONS

In this study, the comparative analysis of AHP and Fuzzy AHP is presented for the multicriteria inventory classification. Using Analytic Hierarchy Process, the normalized weight of each attributes is depicted which in shown in Figure 3, which shows that annual demand has higher priority (0.273) than the other criteria. Fuzzy linguistic terms has been employed for facilitating the comparisons between the subject criteria, since the decision makers feel much comfortable with using linguistic terms rather than providing exact crisp judgments. Using Chang’s extent analysis, the normalized weight of each attributes is depicted which in shown in Figure 4, which shows that still annual demand has higher priority (0.273) than the other criteria.

The results of the comparative analysis of AHP and Fuzzy AHP are given in Table 8. Using AHP and Fuzzy AHP model, the control scheme or appropriate class of 351 raw materials of switch gear section have been determined. It can be noted that different items belongs to different classes or control scheme using different models. Here, this is the point that should not be missed, classical and fuzzy methods are not the competitors with each other at same conditions. The important point is that if the information/evaluations are certain, classical method should be preferred; if the information/evaluations are not certain, fuzzy method should be preferred. In recent years, because of the characteristics of information and decision makers, probable
deviation should be integrated to the decision making processes, and because of that for each decision making method, a fuzzy version is developed. Fuzzy AHP method is a natural result of this necessity.

Table 8: Comparative analysis of AHP and Fuzzy AHP for Multicriteria Inventory Classification

<table>
<thead>
<tr>
<th>Model</th>
<th>Class A</th>
<th>Class B</th>
<th>Class C</th>
<th>Total items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>very important or</td>
<td>Important or</td>
<td>Unimportant or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>outstandingly important</td>
<td>average important</td>
<td>relatively unimportant</td>
<td></td>
</tr>
<tr>
<td>AHP</td>
<td>22</td>
<td>45</td>
<td>284</td>
<td>351</td>
</tr>
<tr>
<td>Fuzzy AHP</td>
<td>22</td>
<td>47</td>
<td>282</td>
<td></td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

In today’s manufacturing and business environment, an organization must maintain an appropriate balance between critical stock-outs and inventory holding costs. Because customer service is not a principal factor for attracting new customers, but it is frequently a major reason for losing them. Many researchers have devoted themselves to achieving this appropriate balance. Multi-class classification utilizing multiple criteria requires techniques capable of providing accurate classification and processing a large number of inventory items. In this research, a comparative analysis of Analytic Hierarchy Process (FAHP) and Fuzzy Analytic Hierarchy Process (FAHP) for multi-criteria inventory classification model has been presented.

AHP is one of the most commonly used techniques when decision problems contain social, economic, technical and political factors that need to be evaluated by linguistic variables. Multicriteria decision making techniques based on the linguistic evaluations like AHP helps to make a best selection decision by using a weighting process within the current alternatives via pair wise comparisons. But for the uncertain or fuzzy environment, fuzzy numbers have to use for the evaluation due to the deviations of decision makers. The FAHP approach proved to be a convenient method in tackling practical multi-criteria decision making problems. Fuzzy AHP technique was used to synthesize the opinions of the decision makers to identify the weight of each criterion. It demonstrated the advantage of being able to capture the vagueness of human thinking and to aid in solving the research problem through a structured manner and a simple process.

The classification systems were very flexible in the sense that the user:

- can incorporate some other criteria or remove any criteria for his/her specific implementation;
- can conduct different classification analyses for different inventory records;
- can employ an application-specific linguistic variable set;
- can substitute the crisp comparison values $a_{ij}$ for the fuzzy comparison values $\tilde{a}_{ij}$ in the optimization program, whenever the fuzzy comparisons are not available.

The new inventory classes also suggest the following inventory policies:

Class-A:

1. Since the main inventory investment is devoted to class- A items, the tightest controlling and auditing levels should be applied to this class.
2. Since most of the items clustered in this class were deemed as ‘critical’, detailed demand forecast reports should be prepared and appropriate levels of safety stock should be maintained.
(3) Detailed recording systems should be established (i.e. via automated records, barcode systems, etc.).

Class-B:

(1) A periodic review policy may be adopted, less detailed forecasts and simpler purchasing procedures can be employed (i.e. economic order quantity may be used).

(2) Rational levels of safety stock can be held for the class-altering items those were clustered in class-A with a traditional ABC analysis.

Class-C:

(1) Some of the control and record processes may be ignored.

(2) Review periods may be irregularly scheduled.

REFERENCES


For Multicriteria inventory classification, Questions were designed to elicit judgments about the relative importance of each of the selected criteria of a Power Engineering Company in Bangladesh (Energypac Engineering Limited).

For multiple criteria inventory classification, five criteria were included. All the criteria are positively related to the importance level. The criteria are as follows:

1. **Unit Price**
2. **Annual Demand**
3. **Criticality**
4. **Last Used**
5. **Durability**

**APPENDIX A**

**Questionnaire for Multicriteria Inventory Classification**

For Multicriteria inventory classification, Questions were designed to elicit judgments about the relative importance of each of the selected criteria of a Power Engineering Company in Bangladesh (Energypac Engineering Limited).
Table for preference weight values for different level of importance for AHP model.

<table>
<thead>
<tr>
<th>Importance/weights</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Weak importance of one over another</td>
<td>Experience and judgment slightly favor one activity over another</td>
</tr>
<tr>
<td>5</td>
<td>Essential or strong importance</td>
<td>Experience and judgment strongly or essentially favor one activity over another</td>
</tr>
<tr>
<td>7</td>
<td>Demonstrated or confirmed importance</td>
<td>An activity is strongly favored over another and its dominance demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Extreme or Absolute importance</td>
<td>The evidence favoring one activity over another is of the highest degree possible of affirmation</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values between the two adjacent judgments</td>
<td>Used to represent compromise between the preferences listed above</td>
</tr>
</tbody>
</table>

Sample Responses:

____3____ Unit Price: Annual Demand ________

** For inventory classification unit price is slightly important than annual demand **

Judgments matrix for attributes using AHP

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Unit Price</th>
<th>Annual Demand</th>
<th>Criticality</th>
<th>Last Used</th>
<th>Durability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Price</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual Demand</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criticality</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last Used</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Durability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table for preference weight values for different level of importance for Fuzzy AHP model.

<table>
<thead>
<tr>
<th>Linguistic scale for importance</th>
<th>Fuzzy numbers</th>
<th>Membership function</th>
<th>Domain</th>
<th>Triangular fuzzy scale (l, m, u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Just equal</td>
<td>1</td>
<td>( \mu_M(x) = (3-x) / (3-1) )</td>
<td>1 ≤ x ≤ 3</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Equally important</td>
<td>3</td>
<td>( \mu_M(x) = (5-x) / (5-3) )</td>
<td>3 ≤ x ≤ 5</td>
<td>(1, 3, 5)</td>
</tr>
<tr>
<td>Weakly important</td>
<td>7</td>
<td>( \mu_M(x) = (9-x) / (9-7) )</td>
<td>7 ≤ x ≤ 9</td>
<td>(5, 7, 9)</td>
</tr>
<tr>
<td>Essential or Strongly important</td>
<td>5</td>
<td>( \mu_M(x) = (3-1) / (3-1) )</td>
<td>1 ≤ x ≤ 3</td>
<td>(1, 1, 3)</td>
</tr>
<tr>
<td>Very strongly important</td>
<td>7</td>
<td>( \mu_M(x) = (7-x) / (7-5) )</td>
<td>5 ≤ x ≤ 7</td>
<td>(3, 5, 7)</td>
</tr>
<tr>
<td>Extremely Preferred</td>
<td>9</td>
<td>( \mu_M(x) = (9-x) / (9-7) )</td>
<td>7 ≤ x ≤ 9</td>
<td>(5, 7, 9)</td>
</tr>
</tbody>
</table>

If factor \( i \) has one of the above numbers assigned to it when compared to factor \( j \), then \( j \) has the reciprocal value when compared to \( i \) Reciprocals of above \( M_i^{-1} = (1/l_i,u_i,m_i,1/l_i) \)

Judgments matrix for attributes using Fuzzy AHP
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<table>
<thead>
<tr>
<th>Attributes</th>
<th>Unit Price</th>
<th>Annual Demand</th>
<th>Criticality</th>
<th>Last Used</th>
<th>Durability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Price</td>
<td>1,1,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual Demand</td>
<td></td>
<td>1,1,1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Criticality</td>
<td></td>
<td></td>
<td>1,1,1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last Used</td>
<td></td>
<td></td>
<td></td>
<td>1,1,1</td>
<td></td>
</tr>
<tr>
<td>Durability</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1,1,1</td>
</tr>
</tbody>
</table>