PREDICTIVE EVALUATION OF THE STOCK PORTFOLIO PERFORMANCE USING FUZZY C-MEANS ALGORITHM AND FUZZY TRANSFORM

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ABSTRACT

The aim of this paper is to investigate the trend of the return of a portfolio formed randomly or for any specific technique. The approach is made using two techniques fuzzy: fuzzy c-means (FCM) algorithm and the fuzzy transform, where the rules used at fuzzy transform arise from the application of the FCM algorithm. The results show that the proposed methodology is able to predict the trend of the return of a stock portfolio, as well as the tendency of the market index. Real data of the financial market are used from 2004 until 2007.

KEYWORDS

FCM Algorithm; Fuzzy Transform; Overreaction; Financial Ratios

1. INTRODUCTION

The development of tools to help decision making in financial market has been the subject of intense research world over for decades. Generally speaking, financial decisions aim at maximizing profit and, in this context, the evolution of the basic premises has given birth to conflicting theories, such as the Theory of Efficient Market and the Theory of Behavioral Finance [1]. Each of these theories was structured on the fundamental contributions of many reputed authors. Included among these are the original works that were awarded the Nobel Prize for Economics, such as the Portfolio Theory proposed by Markowitz [2, 3] and the Theory of Behavioral Finance proposed by Kahneman and Smith [4, 5]. These give a clear idea of the importance of decision making in economic theory.

Summing up, the Theory of Efficient Market assumes that the investor is rational and is, therefore, likely to avoid risk-taking. Consequently, investors’ make their decisions only on the basis of information strongly supported by statistics and probabilities which supposedly project the future performance of the assets, as also of the financial market as a whole. In contrast to this, the Behavioral Theory uses models which take into account psychological factors that tend to influence the investors in decision making. In other words, investors are not wholly rational, because their decisions are affected by their preferences and beliefs associated with heuristics and rule of thumb.

So, financial market, due to its complexity, has been subject of intense research worldwide in the areas of economics and engineering, in order to better understand and explain this system. One of the major challenges in research on the financial market is the formation of a stock portfolio that produces a considerable profit for investors. For this, some models were developed to solve this problem, as the theory of portfolio selection proposed by Markowitz [2], the asset pricing model
proposed by Sharpe [6], models derived from behavioral finance theory [7], [8], [9] among others. Recently, some approaches using artificial intelligence techniques, widely used in engineering, have been used to develop models for the formation of portfolios. As an example, Abiyev Menekay [10] utilize the theory of genetic algorithms for formation of stock portfolios, Mohamed and Mohamad [11] based on a fuzzy approach, develop a method for forming of stocks portfolio. However, the biggest difficulty is to obtain a portfolio that, in the future, produce a positive return for the investor and also predict the value of this return is almost impossible due to large variations in the financial market, which can be viewed as a nonlinear system and time variant. Thus, to form a stocks portfolio and predict whether this portfolio tends to produce a positive return or a negative return results in an information very relevant to investors. In this paper, the main objective is to develop a model which indicates the trend of a stock portfolio formed randomly or by some specific method using, as constructive basis, the fuzzy c-means (FCM) algorithm [12] and fuzzy transform (F-Transform) [13].

The results show that the proposed model indicates the real trend of a stock portfolio with regard to market index and considering the return produced by stocks in the portfolio. The analysis is performed using real data from the fourth quarter of 2004 to the fourth quarter of 2007. A preliminary version of this work was published and presented in IEEE Computational Intelligence for Financial Engineering & Economics 2012 [14]. The paper is organized as follows: basic concepts related to FCM algorithm and fuzzy transform are introduced in section 2; a brief theory on Behavioral Finance is presented in section 3; the methodology employed is described in section 4; section 5 presents numerical results; finally, conclusions are presented in section 6.

2. MATHEMATICAL BACKGROUND

One of the main characteristics of Fuzzy Set Theory, which was first presented by Zadeh in 1965 [15] is that it allows the treatment of problems involving linguistic variables.

Following original idea of [15], a fuzzy set of an universe of objects \( X \) is defined as function \( \mu \) that maps \( X \) into \([0,1]\), i.e., \( \mu : X \rightarrow [0,1] \). When \( X \) is a finite set \( x_1, \ldots, x_n \), a fuzzy set \( A \) on \( X \) is expressed as \( A = (x, \mu_A(x) \mid x \in X) \), where \( \mu_A(x) \) is called the membership function or grade of membership of \( x \) in \( A \).

The Fuzzy Theory is applied in a number of areas, such as Engineering [16], Medicine [17], Biology [18], and Economics [19, 20].

Among several techniques for grouping or classifying of the elements in subsets of a given set, Fuzzy c-Means- FCM algorithm has proved to be an effective tool when the features or attributes of the analyzed elements can be represented by a vector of real numbers. In such cases, the FCM algorithm allows for identifying clusters or groups of elements in such a way that the elements in the same group may be more similar to, or more compatible with, themselves than elements in different groups from a matrix named pattern matrix [12] of dimension \( n \times p \), \( n \) being the number of elements and \( p \) the dimension of the vectors of features of these elements. The membership degree will be defined here on the basis on the Euclidean distance between the elements in a space of dimension \( p \).

As in the specific application of the present paper, the analyzed elements will be grouped into two subsets, the following presentation will be particularized for that situation. Then, the FCM algorithm determines the subsets \( C_1 \) and \( C_2 \) through the solution of the following problem [12].
Given the elements \(x_1, x_2, \ldots, x_n\), described as vectors of dimension \(p\), determine the vectors \(c_1\) and \(c_2\), also of dimension \(p\), and the membership degrees \(\mu_j(x_j) \geq 0\) and \(\mu_j(x_j) \geq 0\), \(j = 1, 2, \ldots, n\), in a way to minimize the function

\[
J(U, c) = \sum_{j=1}^{n} \sum_{i=1}^{m} [\mu_i(x_j)]^2 \|x_j - c_i\|^2
\]  

(1)

applying the condition \(\mu_i(x_j) + \mu_j(x_j) = 1\), \(j = 1, 2, \ldots, n\).

The stronger the proximity of an element to a given subset, that is, the shorter the distance between an element and the center of a given subset, the closer will be the membership degree to the unity of that subset.

The optimization problem given by equation (1) can be solved analytically, and the solution is given by [21]:

\[
c_i = \frac{1}{\sum_{j=1}^{m} [\mu_i(x_j)]^2} \sum_{j=1}^{m} [\mu_i(x_j)]^2 x_j
\]  

(2)

for \(i = 1, 2\)

\[
\mu_i(x_j) = \frac{\left(1 \cdot \frac{1}{\|x_j - c_i\|^2}\right)^{1/(m-1)}}{\sum_{k=1}^{m} \left(1 \cdot \frac{1}{\|x_j - c_i\|^2}\right)^{1/(m-1)}}
\]  

(3)

In equations (1), (2) and (3), \(m\) is called the exponent weight, which reduces the influence of small grade membership (point further away from \(c_i\)) compared to that of large grade membership (point close to \(c_i\)) [12]. According to [12], no theoretical justification exists for choosing \(m\), but it is usually taken that \(m = 2\) and the same is followed here. As one can very well observe, the calculation of the vectors of center \(c_i\) given by equation (2) depends on the membership degrees \(\mu_i(x_j)\). These, in turn, depend on \(c_i\), according to equation (3). The solution can be obtained iteratively by the algorithm named FCM, as described below.

Step 1: Initiate the membership matrix in such a way that \(\mu_i(x_j) + \mu_j(x_j) = 1\), \(j = 1, 2, \ldots, m\) and \(\mu_i(x_j) \geq 0\) and \(\mu_j(x_j) \geq 0\), \(j = 1, 2, \ldots, m\);

Step 2: Calculate the centers \(c_1\) e \(c_2\) by equation (2);

Step 3: Recalculate the new membership matrix via equation (3) by utilizing the centers obtained in step 2;
Repeat steps 2 and 3 until the value of the objective function represented in equation (1) does not decrease any longer according to the adopted precision.

This algorithm, among other things, can be used to generate rules for a fuzzy inference system. In this sense, the rules derived from application of the Fuzzy c- Means (FCM) Algorithm will be used in a new theory based on fuzzy logic called Fuzzy Transform. The Fuzzy transform, developed by Perfilieva [13], takes a function and uses the membership degree of an element with regard to the determined subset to produce a set-to-point correspondence between fuzzy sets from the partition and certain average values of that function.

So, let \( A_1, ..., A_n \) be membership functions on interval \([a, b]\) and a function \( f \) belonging to the set of continuous function on interval \([a, b]\). The n-tuple of real numbers \([F_1, ..., F_n]\) given by (4) is the fuzzy transform of \( f \) with respect to \( A_1, ..., A_n \) [13].

\[
F_K = \frac{\int_a^b f(x)A_K(x)dx}{\int_a^b A_K(x)dx}, \quad K = 1, ..., n \tag{4}
\]

The \( k \)th component of the Fuzzy Transform minimizes the function showed in (5).

\[
\phi(y) = \int_a^b (f(x) - y)^2 A_K(x)dx
\]

where \( y = F_K \)

Once know the Fuzzy Transform components \( F_K \), its possible (approximately) reconstruct the original function \( f \) using (6):

\[
f = \sum_{K=1}^n F_K \cdot A_K(x) \tag{6}
\]

In [22] is demonstrate an application of Fuzzy Transform investigating the dependence of GDP (Gross Domestic Product) with respect to some variables. The goal here is apply the FCM algorithm in some companies in the financial market, analyze the grouping and extract rules that will be used to transform fuzzy. In addition, will be explored the strong connection between fuzzy logic and behavioral finance.

3. Behavioral Finance

At the beginning of 70s, when the Theory of Efficient Market has been greatly influencing decision making in Economics, the search for understanding behavioral abnormalities in financial markets the world over led to the proposition of the concepts of Psychology and Sociology in economic analysis, which eventually gained acceptance to such an extent that the proponents of these concepts—Kaheneman [4] and Smith [5]—were awarded the Nobel Prize of Economics in 2002. The elements involved in this new approach led to the elaboration of the Theory of Behavioral Finance.

According to the Behavioral Theory, individuals deviate from statistical rules by making decisions guided by heuristics, or rules of thumb, as opposed to the Theory of Efficient Market in
the economic context. The Cognitive Psychology which studies the mechanisms of thinking is at the basis of this approach and shows that individuals overvalue their recent experiences, as also the trust in their skills, thus contributing to distortions in their thinking [9]. Several heuristics influence decision making. The next section describes the deviations caused by the use of the heuristics of representativeness and anchorage [23], which are directly related to the Fuzzy Set Theory.

### 3.1. Heuristic of Representativeness

The heuristic of representativeness is associated essentially with the similarity between the analyzed elements. In the sequence, a classical example is presented in which decision making is strongly influenced by descriptive pieces of information, even when the probability of occurrence is known. In other words, individuals prefer to value descriptive information to consider the probability of occurrence of the fact.

In this classical example presented in the literature, some individuals were asked to guess what could be the occupation of a randomly chosen person in a group of ten people, given that eight people in the group were truck drivers and the other two accountants [24]. In the first case, all the ten people were dressed in the same manner and when one of them was chosen, majority of the participants, relying on the known probability, thought that that person was a truck driver. In the second case, however, an element of ambiguity was introduced: the ten people were dressed differently and an individual wearing a suit, glasses and carrying a briefcase was chosen. In this case, majority of the participants identified the chosen person as being an accountant, even though it was known beforehand that the probability of that individual being a truck driver was greater than the probability of his being an accountant.

In this example, the man wearing a suit, glasses and carrying a briefcase had more similarities with the set of accountants and, therefore, less similarity with the set of truck drivers. Experiments of this kind strengthen the hypothesis that a method which makes use of fuzzy sets is more efficient than statistical methods in shaping decision making under conditions of ambiguity [25].

In the context of decision making in Economics, individuals under the influence of the heuristic of representativeness tend to produce extreme predictions or overreaction [26], in which former losers tend to be winners in the future and vice-versa [27]. In other words, the stock market overreaction maintains that a given stock decreases or increases too far in price as a consequence of recent bad or good news associated with the stock. Thus, traders who are not sure about the intrinsic value of a stock will be too optimistic about its value when the company is winning, and too pessimistic when it is losing [28, 29].

### 3.2. Heuristic of Anchorage

People often rely on elements or conditions of reference to make decisions. It is said, in this case, that the decision is anchored on a referential, that is, the decision is made on the basis of a heuristic of anchoring. The heuristic of anchoring, as distinct from the heuristic of representativeness, leads to excessive moderation in decision making, causing underreaction phenomenon [26] in which former winners tend to be future winners, and former losers tend to be future losers [27]. It is associated with conservative decision making, restraining people from quick changes in their beliefs based on new information. An experiment conducted in [24] shows the influence of anchoring heuristic on the decision of an individual. In this experiment, two groups of students were asked to estimate the value of an expression in five seconds:
Although the correct answer to the two sequences is 40,320, the average estimate given for sequence 2 was 512, and that given by group 1 (descending sequence) 2,250. This is because, in the descending sequence, the first steps of multiplication (from left to right) are equal to higher numbers. In terms of the Fuzzy Set Theory, a decision based on this heuristic is focused on the most representative element of the set, that is, the element of total membership, $\mu(x) = 1$ [23]. The FCM algorithm intrinsically possesses the representativeness and anchoring heuristics arising of Behavioral Finance Theory, as established in [23], [32].

4. METHODOLOGY

This section presents the methodology developed to predict the trend of the return of a portfolio formed randomly or by some specific technique. The proposed methodology for obtaining the model consists of two bases constructive, where the analysis was done quarterly from 2004 to 2007. In this first base, the goal is obtain two clusters formed by stocks of financial market, a good cluster and bad cluster. The data or features of the stocks utilized by the model are financial ratios divulged periodically by public companies, comprehending some profitability ratios, market indicators and debt-equity ratio.

The relation between these ratios and the financial return of the stocks is a highly discussed theme in literature [30, 31]. Several sets of financial ratios, related to marketability, profitability, indebtedness and stocks rating have been tested and grouped in different ways for the development of this paper. The ratios selected and effectively adopted here have produced the best results and are divided thus:

- Profitability ratios: net profit margin, return on net worth, return on assets [30];
- Debt ratio: debt to equity [30];
- Market indicators: price on book value of equity, price on earning per share [31].

The following are the two steps of the proposed model:

Step 1: In this step, called pattern recognition, the fuzzy c-means algorithm is applied to a given set of stocks with financial ratios divulged for a certain period $t$, producing two subsets associated with that period. The basis for the algorithm is a pattern matrix $n \times p$, in which each line corresponds to one company of the set of considered stocks and each column to the financial ratios associated with the company of that set. According to section 2, each subset results, characterized by a vector of center, and the stocks classified in this group are those with higher membership degrees when compared to the membership degrees related to the stocks of the other group. This membership degree indicates similarity between the stocks and each cluster. Stocks with membership degree inferior to 0.65 in regard to cluster are discarded, because a membership degree equal to 0.65 indicates equivalent similarity of a stock in regard to each cluster.

For each group in the period $t$, the average financial return at the end of period $t+1$ may be calculated thus:

$$ r_{t+1} = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{P_{t+1}^{i}}{P_{t}^{i}} $$

where
The group with a greater average financial return is called the good group and the one with a smaller average financial return the bad group. Each group is represented by a vector of financial ratios calculated iteratively using equations (2) and (3) until the objective function is minimized.

Thus, going by the definition of the anchoring heuristic, in which the estimates are formed from an initial value to produce the final answer, the calculation of these vectors of financial ratios is based on the anchoring heuristic. It is interesting to notice that classification of the groups, as good or bad, has been possible only at the end of period $t+1$, that is, this classification is \textit{a posteriori}.

The objective of step 2 is to set a procedure that allows to classify, at the end of period $t$, the group which supposedly will have a good or a bad performance at the end of period $t+1$. For the developments of step 2, each period will be considered as a quarter of the year. The reason behind this assumption is that the financial ratios used for classification in the numerical application of the proposed model are divulged at the end of each quarter of the year.

Step 2: This step, called rating of stocks, aims at classifying, at the end of quarter $t$, stocks whose performance will be supposedly good or bad by the end of the next quarter, $t+1$. Thus, unlike in step 1, the aim here is to set a classification \textit{a priori}.

With this purpose, let the set of good and bad centers corresponding to the 1$^{st}$ quarters be obtained in step 1 for some years. The FCM algorithm can obviously be applied to these centers, thus producing two new reference centers. Such centers, which are actually centers of sets of centers, will be called winner and loser centers; the centers around which there is a greater number of good centers are called the winner centers, and those around which there is a greater number of loser centers the bad centers. Both the winner and loser centers are assumed as respective references for the promising and non-promising stocks to be classified at the end of every 1$^{st}$ quarter. A similar procedure is adopted for the sets of 2$^{nd}$ quarters, 3$^{rd}$ quarters and 4$^{th}$ quarters.

Therefore, for the classification of a stock in a particular quarter of the year, it is enough to calculate the membership degrees related to the winner and loser centers corresponding to that quarter. The higher membership degree with respect to the winner center defines the stock as being promising, and the one with respect to loser center as being non-promising in that quarter. In what follows, for each quarter of the year, the group of promising stocks will be called winner portfolio and the group of non-promising stocks loser portfolio. Although the winner and loser portfolios may be formed at the end of each quarter, evaluation of their performance will be possible evidently at the end of the next quarter only.

The assets in each group are classified depending on the degree of similarity which is calculated by equation 3. The greater the degree of similarity, the shorter the distance between the center vector and the ratio vectors of the asset. In this sense, there is an evident link between the proposed methodology for classification and the representativeness heuristic, which is based on the degree of similarity that an object $A$ belongs to class $B$. 

\[ P_t^i \] - value of stock $i$ at the end of period $t$, 
\[ P_{t+1}^i \] - value of the stock $i$ at the end of period $t+1$, 
\[ n \] - number of stocks classified in the group.
More details on application of the methodology presented and of the results obtained in this first base, can be found in [7, 32], where was established that loser portfolios (portfolio with low financial return in the quarter $t$) presents in the quarter $t+1$ a return greater than the winner portfolio (portfolio with high financial return in the quarter $t$). The results obtained in [32] have some notable implications that are discussed following.

A. Main Findings

The application of this methodology suggests that overreaction does exist in the American Stock Market, and that an investor can obtain abnormal profits with a systematic application of the contrarian strategy; in other words, the investor can obtain abnormal profits with short selling of the winner portfolio and purchasing of a loser portfolio. In finance, short selling is the practice of selling assets that have been borrowed from a third party (usually a broker) with the intention of buying identical assets at a later date to return to the lender. The short seller hopes to profit from the decline in the price of the assets between the sale and the repurchase, as the seller will pay less to buy the assets than what he or she received when selling them.

An example: if shares in company XY are currently traded at 50 dollars per share, a short seller can borrow 100 shares of that company and immediately sell them for a total of 5000 dollars. If the price of the shares later declines to 40 dollars per share, the short seller can buy back 100 shares for 4000 dollars, return the shares to the lender and keep the 1000 dollars profit (minus borrowing fees).

The results obtained by applying the behavioral fuzzy model in the American stock market confirm the findings of [33], thus supporting the overreaction hypothesis in the American market. In general, the results are consistent with the overreaction phenomenon and, consequently, with the application of the contrarian strategy in the American stock market.

B. Implications of Other Empirical Work

The results of this paper have interesting implications for previous work with regard to $P/E$ ratio effects, and Efficient Market Hypothesis which claims that prices "fully reflect" available information [34]. With regard to available information, the market may be tested if it conforms to any of the following forms: weak form, in which the information set is just historical price; semi-strong form, in which the concern is whether prices efficiently adjusted to other information that is publicly available (announcements of annual earnings, financial ratios, and so on); strong form, concerned with whether the given investors or groups have monopolistic access to any relevant information for price formation. Indirectly, the methodology used in this paper tests the American stock market in the semi-strong form, suggesting overreaction in this market and indicating that the American market is informationally inefficient in the semi-strong form, once there is significant evidence of overreaction.

With regard to the $P/E$ effect, [35] finds that stocks with high $P/E$ ratios generate lower returns and, those with low $P/E$ ratios higher returns. In [36] was detected mean-reverting behavior in stock performance when portfolios are sorted on the basis of observed $P/E$ ratios. However, in this paper a relation apparently exists between the $P/E$ ratio and debt to equity ratio. The methodology applied in the American market produces a winner center vector and a loser center vector, consisting of the six financial ratios, which are used for the generation of portfolios. Once there is significant evidence of overreaction, the loser center generates a portfolio with higher return in the future.
As shown in figures 1, 2 and 3, there seems to be an inverse relation between P/E ratio and the debt to equity ratio, because high (low) \( P/E \) ratio is associated with low (high) debt to equity ratio for all quarters and for all analyzed sectors.

![Figure 1. P/L versus Net Debt – Oil and Gas Sector](image1)

![Figure 2. P/L versus Net Debt – Textile Sector](image2)
In all quarters, with regard to loser center vector which produces a portfolio with higher return, not always the $P/E$ ratio is low; rather, the low $P/E$ ratio is associated with a high debt to equity ratio, suggesting that $P/E$ ratio should not be analyzed separately. In this case, the overreaction is associated with other ratios, as debt to equity ratio.

It’s emphasized that [27] claims that apparent anomalies can be due to methodology, and anomalies tend to disappear with reasonable changes in technique. So, the model here presented is an auxiliary toll which, together with other models, aims to better understand this complex system, nonlinear and time variant that is the financial market. Thus, these findings strengthen the overreaction hypothesis in American Stock Market. Briefly, the first base provides the following information:

- There is significant evidence of overreaction in the American stock market;

- There is a relationship between the price earnings ratio and and net debt ratio. A high net debt ratio associated with a low price earnings ratio generates a loser portfolio with high return in future.

So, at second base were considered only two ratios, the P/E ratio and debt ratio figure 4 illustrates the basic methodology developed in this second constructive basis and figure 5 represents the block Fuzzy Transform Model contained in figure 4.
In figure 5, the inputs are represented only by two features of the assets: P/E and debt ratios, and the output is the estimate return produced by the set of stocks chosen. The fuzzy transform, in turn, estimates the average return of all stock used, based on IF-THEN rules originated in the first basis. In other words, these rules, which manage an estimated of the return in function of the inputs, derived from information contained in the first constructive basis.

5. RESULTS

The results here presented were obtained using real data of the U.S. stock market. To characterize the assets, were used just two market ratios: the price earnings ratio and net debt ratio. The reason for using only these two ratios to characterize an asset arise from the observation described in the section 4; in other words, the price earnings ratio is the focus of study in overreaction analysis in the stock market [33], [35], however, was observed in [32] that exists a relationship between the price earnings ratio and net debt ratio with regard to overreaction in the stock market. So, considering this relationship and that there overreaction in the U.S. stock market [32], thirteen rules were formed considering price earnings ratio and the net debt ratio as a condition, and as consequently the financial return of the asset. The assets of a stock portfolio formed randomly in a quarter \( t \), were inserted into the fuzzy transform that producing the return of each asset in question. Soon after, the average return was calculated and compared with the real average return of the portfolio in a quarter \( t+1 \). Figure 6 shows the result of this comparison from the first quarter of 2005 until the fourth quarter of 2007.
Comparing the average return of the portfolio fuzzy transform with the return produced by the S & P500 index, were obtained the results showed in the figure 7.

The results show that, in most cases, when comparing the return predicted by the model with the actual return of the same portfolio, the proposed model proves to be very accurate in indicating the true trend of a portfolio forming randomly or by a technique specific. In some cases, the predicted return is close to the actual return, as can be seen in figure 6. Making the same comparison with the S&P500, the proposed model is efficient in predicting the real trend of this market index, as shown in figure 7.

6. CONCLUSION

The model here proposed is the result of the fusion of two techniques: a clustering algorithm known as Fuzzy c- Means (FCM) and the Fuzzy Transform (F- Transform). The rules used in the Fuzzy Transform are produced by FCM algorithm used in the first constructive basis of the
model, in other words, the information contained in the first base of the model were used at second constructive base to produce the final result, which is to obtain the trend of high or low of the return of a stock portfolio. The results suggest that the proposed model is able of predict a high or low of the return of a stock portfolio formed randomly or by some specific technique. Furthermore, the results suggest that the model has the property of predict the tendency of the market index.

Some contributions can still be observed:

i) The financial is a very complex system, time- varying and dependent of qualitative and quantitative information. In this sense, some models are required to define more precisely the dynamics of this system. In the case of portfolios formation, there are several models in order to form a portfolio with a good performance in the future. However, to increase the confidence of the investor in the results produced by such model, would be very helpful to confirm the trend of this portfolio through another model. The methodology proposed here has this goal.

ii) In the practice there are many lay investors that form a portfolio randomly picking stocks of companies that they trust or stocks of companies that appear to be healthy. With the methodology proposed here, it’s possible to find with precision, the real trend of the portfolio formed randomly.

Since the financial market is a highly complex system and subject to various interferences, the results produced by model become informationally useful to an investor, in addition to being an auxiliary tool for analysis of a stock portfolio.

7. References


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