A NEW ANALYSIS OF FAILURE MODES AND EFFECTS BY FUZZY TODIM WITH USING FUZZY TIME FUNCTION

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ABSTRACT

Failure mode and effects analysis (FMEA) is a widely used engineering technique for designing, identifying and eliminating the known and/or potential failures, problems, and errors and so on from system to other parts. The evaluating of FMEA parameters is a challenging point because it’s important for managers to know the real risk in their systems. In this study, we used fuzzy TODIM for evaluating the potential failure modes in our system respect to factors of FMEA, which is known as: Severity (S); Occurrence (O); and detect ability (D). The final result was combined with fuzzy time function which helps to predict systems in future and solving problems in our system and it could help to avoid potential future failure modes in our systems.

KEYWORDS

Fuzzy FMEA, Fuzzy set theory, Fuzzy TODIM, Fuzzy time function.

1. INTRODUCTION

Failure mode and effects is one of the subjects in organization and factories which can help decreasing the cost and improving effectiveness in system. FMEA, provides a framework or cause and effect analysis of potential product failures. Chin, Chan& Yang, 2008 [1] has a purpose of prioritizing the risk priority number (RPN) of the product design or planning process to assign the limited resources to the most serious risk item. FMEA, designed to provide information for risk management formal design methodology by NASA in 1963 for their obvious reliability requirements and then, it was adopted and promoted by Ford Motor in 1977 (Chin et al, 2008)[1]. Since then, it has become a powerful tool extensively used for safety and reliability analysis of products and process in a wide range of industries especially aerospace, nuclear and automotive industries (Gilchrist, 1993 [2]; Sharma, Kumar 2005 [3]) and these kinds of research shows the impotency of FMEA in industrials.
Ben-Daya & Raouf, (1996)[4], each failure mode can be evaluated by three factors as severity, likelihood of occurrence, and the difficulty of detection of the failure mode. In a typical FMEA evaluation, a number between 1 and 10 (with 1 being the best and 10 being the worst case) is given for each of the three factors. By multiplying the values for severity (S), occurrence (O), and detectability (D), a risk priority number (RPN) is obtained, which is RPN = S × O × D (Chin et al, 2008). Then the RPN value for each failure mode is ranked to find out the failures with higher risks.

The base values of RPNs have been considerably criticized for many reasons, most of them are stated below:

- The relative importance among the three risk factors occurrence, severity, and detection is not considered as they are accepted equally important.
- Different combination of O, S and D may produce exactly the same value of RPN, although their hidden risk implications may be totally different. For instance, two different failures with the O, S and D values of 4, 3, 3 and 9, 1, 3, respectively, have the same RPN value of 36.
- The use of multiplication method in the calculation of RPN is questionable and strongly sensitive to variations in criticality factor evaluations.

When the traditional FMEA and the fuzzy approach are compared, the fuzzy approach has an advantage of allowing the conduction of risk evaluation and prioritization based on the knowledge of the experts (Tay & Lim 2006) [5].

Xu, Tang, Xie, Ho, and Zhu (2002) [6], state the reasons for considering the fuzzy logic approach as following:

- All FMEA-related information is taken in natural language which is easy and plausible for fuzzy logic to deal with as it is based on human language and can be built on top of the experience of experts.
- Fuzzy logic allows imprecise data usage so it enables the treatment of many states.

In this study we collect data from experts with fuzzy linguistic numbers, this data shows each potential failure modes in our systems respect to the three risk factors of FMEA, and then we used fuzzy TODIM for ranking this alternatives and factors of FMEA. In final steps we used fuzzy time function for considering time in our decisions making for future, and with combination of fuzzy time function and Todim, the final results shown in our study.

2. METHODOLOGY

2.1. Fuzzy FMEA

Significant efforts have been made in FMEA literature to overcome the shortcomings of the traditional RPN, Wang et al, 2009[7]. The studies about FMEA considering fuzzy approach use the experts who describe the risk factors O, S, and D by using the fuzzy linguistic terms. The linguistic variables were used for evaluating the risk factors against the alternative. This helps to reduce calculations and do the same way Fuzzy AHP and TOPSIS Fuzzy do with this benefit to
determine in one step. The linguistic variables as an interpretation of traditional FMEA are in 10 point scale. (1-10)

<table>
<thead>
<tr>
<th>Criteria</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Severity</td>
<td>S</td>
</tr>
<tr>
<td>occurrence</td>
<td>O</td>
</tr>
<tr>
<td>Detect ability</td>
<td>D</td>
</tr>
</tbody>
</table>

Table 1. Criteria

2.2. Fuzzy Logic

A fuzzy set is a class of objects with grades of membership. A membership function is between zero and one, Zadeh, 1965. [8]. Fuzzy logic is derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. It allows the model to easily incorporate various subject experts’ advice in developing critical parameter estimates, Zimmermann, 2001[9]. In other words; fuzzy logic enables us to handle uncertainty.

There are some kinds of fuzzy numbers. Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. It is represented with three points as below: a = (a_1, a_2, a_3). The membership function is illustrated in Eq. (1).

Let A and B are defined as a = (a_1, a_2,a_3), b= (b_1,b_2,b_3). Then c= (a_1+b_1,a_2+b_2,a_3+b_3) is the addition of these two numbers. Besides, d= (a_1-b_1,a_2-b_2,a_3-b_3) is the subtraction of them. Moreover, d= (a_1.b_1,a_2.b_2,a_3.b_3) is the multiplication of them (Klir& Yean, 1995[10]; Lai & Hwang; 1995[11]; Zimmermann, 2001[9]).

Ahmet et al.2012[12], used triangular fuzzy number to collect data, and then used fuzzy Topsis to evaluate data which we used here interval valued fuzzy number and TODIM, and we get better results.

This study use interval-valued triangular fuzzy for collecting expert’s opinion (Table .2)

<table>
<thead>
<tr>
<th>Linguistic preferences</th>
<th>Interval-valued TFNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very poor</td>
<td>[(0,0),0,(1.1.5)]</td>
</tr>
<tr>
<td>Poor</td>
<td>[(0,0.5),1,(2.5,3.5)]</td>
</tr>
<tr>
<td>Medium poor</td>
<td>[(0.1.5),3,(4.5,5.5)]</td>
</tr>
<tr>
<td>Fair</td>
<td>[(2.5,3.5),5,(6.5,7.5)</td>
</tr>
<tr>
<td>Medium good</td>
<td>[(4.5,5.5),7,(8.9,5)]</td>
</tr>
<tr>
<td>Good</td>
<td>[(5.5,7.5),9,(9.5,10)]</td>
</tr>
<tr>
<td>Very good</td>
<td>[(8.5,9.5),10,(10,10)]</td>
</tr>
</tbody>
</table>

Table 2. Corresponding TFNS to linguistic preferences.

Definition 1. A TFN a is defined by a triangular with membership function:
Definition 2. The distance between a and b is:

\[
\delta(a + b) = \left\{ \begin{array}{ll}
0, & x < a_i \\
\frac{x - a_i}{a_2 - a_1} a_2 \geq x \geq a_i \\
\frac{a_3 - x}{a_3 - a_2} a_2 \geq x \geq a_3 \\
0, & \text{otherwise}
\end{array} \right. 
\]

Eq.(1)

The TFN is based on a three-value judgment: the minimum possible value \(a_1\), the mean value \(a_2\) and the maximum possible value \(a_3\). The criteria values depend on linguistic preferences.

The weight vector \(w = (w_1, w_2, w_3)\) of the three factors of FMEA is unknown but satisfies \(w_j \geq 0\), \(j = 1, 2, 3\) with \(\sum w_j = 1\). Suppose that the each factor (S,O,D) is denoted \(c_{ij}\) (\(S_{ij} , O_{ij} , D_{ij}\)). Then \(C = [c_{ij}]_{m \times n}\) is a fuzzy decision matrix. In Fig 1, \(c_{ij}\) is expressed in an interval-value TFN:

\[
C = \begin{bmatrix}
(a_1, a_2, a_3) \\
(a_1, a_2, a_3)
\end{bmatrix}
\]

(3)

Then, \(C = [(a_1, a_{1i}); (a_2, a_{3i})]\). The normalized decision matrix \(R\) can thus be calculated. Given \(C_{ij} = [(a_1, a_{1i}); (a_2, a_{3i})]\), the normalized performance rating is:

\[
r_{ij} = \left[ \left( \frac{a_{1i}}{d_j}, \frac{a_{1i}}{d_j}; \frac{a_{2i}}{d_j}, \frac{a_{2i}}{d_j} \right) \right], \quad i = 1, 2, ..., m; \\
\]

\(j = 1, 2, ..., n\) for \(j \in I\) 

(4)

\[
r_{ij} = \left[ \left( \frac{a_{1i}}{d_j}, \frac{a_{1i}}{d_j}; \frac{a_{3i}}{d_j}, \frac{a_{3i}}{d_j} \right) \right], \quad i = 1, 2, ..., m; \\
\]

\(j = 1, 2, ..., n\) for \(j \in J\) 

(5)

Where \(d^*_j = \max \{c_{ji}, i = 1, ..., m\}\) and \(a_j = \min \{a_{ji}, i = 1, ..., m\}\). Given that \(r_j = [(l_{ij}, l_{ij}); (u_{ij}, u_{ij})]\), \(R = [r_{ij}]_{m \times n}\) can be obtained, and \(R_0 = (r_{01}, r_{02}, ..., r_{0n}) = [(1, 1); (1, 1), [(1, 1); 1; (1, 1)]; (1, 1)]\).
The distance between the reference value and each comparison value can be calculated using definition (2):

\[\delta_{ij}^{(1)} = \frac{1}{3} \left[ \left( l_{ij} - 1 \right)^2 + \left( m_{ij} - 1 \right)^2 + \left( u_{ij} - 1 \right)^2 \right]^{\frac{1}{2}}\] (6)

\[\delta_{ij}^{(2)} = \frac{1}{3} \left[ \left( l_{ij} - 1 \right)^2 + \left( m_{ij} - 1 \right)^2 + \left( u_{ij} - 1 \right)^2 \right]^{\frac{1}{2}}\]

This calculation is used to determine the distance between the reference value and the comparison value in the interval \(\delta_{ij} = [\delta_{ij}^{(1)}, \delta_{ij}^{(2)}]\).

Hence this study uses a formula to determine the factor weight (Zhan et al. 2011[13])

\[w_j = \frac{\sum_{i=1}^{n} (\delta_{ij}^{(1)} + \delta_{ij}^{(2)})}{\sum_{i=1}^{n} \sum_{j=1}^{m} (\delta_{ij}^{(1)} + \delta_{ij}^{(2)})}\] (7)

The weight vector of the criteria is applied to decision matrix A. The reference series and the comparison series constitute the interval value \(\delta_{ij} = [\delta_{ij}^{(1)}, \delta_{ij}^{(2)}]\). The concept of a linguistic preference is useful to address uncertainties, i.e., for description in conventional linguistic expression table 2 indicates how the interval-valued triangular fuzzy membership function can accommodate the qualitative data while the evaluators process the evaluation in the form of linguistic information.

2.3. Prospect theory

The value function used in the prospect theory is described in form of a power law expressed as:
\begin{equation}
  v(x) = \begin{cases} 
    x^\alpha & \text{if } x \geq 0 \\
    -\theta(-x)^\beta & \text{if } x < 0 
  \end{cases} \tag{8}
\end{equation}

Where \( \alpha \) and \( \beta \) are parameters related to gains and losses, respectively. The parameter \( \theta \) represents a characteristic that is steeper for losses than for gains when considering cases for which risk aversion \( \theta > 1 \). Fig 2

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Value function of the TODIM method.}
\end{figure}

### 2.4. TODIM

The TODIM method uses paired comparison between the factors by using technically simple resource to eliminate occasional inconsistencies resulting from these comparisons. TODIM allows value judgments to be performed on a verbal scale using a criteria hierarchy, fuzzy value judgments and interdependence relationships among the alternatives. The alternatives (potential risk) \( (A_1, A_2, \ldots, A_m) \) are viable alternatives, \( c_1, c_2, c_3 \) are represented factors, and \( x_{ij} \) indicates the rating of alternative \( A_j \) according to the criteria \( c_i \). The weight vector \( w = (w_1, w_2, \ldots, w_n) \) comprises the individual weights \( w_j \) for each criterion \( c_j \) satisfying \( \sum_{i=1}^{n} w_j = 1 \). The data of the decision matrix \( A \) originate from different sources. The matrix is required to normalize it to transform it into a dimensionless matrix and allows various criteria to be compared. This study use the normalized decision matrix \( R = [r_{ij}]_{m \times n} \) with \( i = 1, \ldots, m \) and \( j = 1, \ldots, n \). M.-L. Tseng et al. (2012) [14]

\begin{equation}
  A = \begin{pmatrix}
    x_{11} & \cdots & x_{1n} \\
    \vdots & \ddots & \vdots \\
    x_{m1} & \cdots & x_{mn}
  \end{pmatrix} \tag{9}
\end{equation}

TODIM then calculates the partial dominance matrices and the final dominance matrix. The first calculation that decision makers must define is a reference criterion (typically the criterion with...
the greatest importance weight). Therefore, \( w_{rc} \) indicates the weight of the criterion \( c \) by the reference criteria \( r \). TODIM is expressed by the following equations. The dominance of each alternative over each alternative is:

\[
\delta(A_i, A_j) = \sum_{c=1}^{m} \phi_c(A_i, A_j)_{\psi(i,j)} \quad (10)
\]

Where

\[
\phi(A_i, A_j) = \begin{cases} 
\frac{w_c(x_u - x_w)}{\sum_{c} w_c} & \text{if } (x_u - x_w) > 0 \\
0 & \text{if } (x_u - x_w) = 0 \\
-1 & \frac{\sum_{c} w_c(x_u - x_w)}{\sum_{c} w_c} & \text{if } (x_u - x_w) < 0 \\
\frac{\sum_{c} w_c(x_u - x_w)}{\sum_{c} w_c} & \theta \end{cases}
\]

The term \( \phi_c(A_i, A_j) \) represents the contribution of criterion \( c \) (c=1,..., m) to the function \( \delta(A_i, A_j) \) when comparing alternative \( i \) with alternative \( j \). The parameter \( \theta \) represents the attenuation factor of the losses, whose mitigation depends on a specific problem. A positive \( (x_u - x_w) \) represents a gain. Whereas a nil or a negative \( (x_u - x_w) \) represents a loss. The final matrix of dominance is obtained by summing the partial matrices of dominance for each criterion. The global value of the alternative \( I \) is determined by normalizing the final matrix of dominance according the following expression:

\[
\xi_I = \frac{\sum_{j=1}^{n} \delta(i, j) - \min_{j=1}^{n} \sum_{j=1}^{n} \delta(i, j)}{\max_{j=1}^{n} \sum_{j=1}^{n} \delta(i, j) - \min_{j=1}^{n} \sum_{j=1}^{n} \delta(i, j)} \quad (12)
\]

Ordering the values \( \xi_I \) provides the rank of each alternative, and better alternatives have higher values of \( \xi_I \). Use of numerical values in rating alternatives may be limited in their capacity to address uncertainties. Therefore, an extension of TODIM is proposed to solve problems with decision making with uncertain data resulting in fuzzy TODIM. In practical applications, the triangular shape of the membership function is often used to represent fuzzy numbers. Fuzzy models using TFNs proved highly effective for solving decision making problems for which the available information is imprecise. Hence, this study provides some basic definitions of fuzzy set theory.
3.2. Fuzzy time function

A fuzzy time function consists of three functions for each way, for example when we are optimistic about some criteria we predict the situation in the best possible way, we could produce a function with collecting the opinion of decision makers, and for normal and pessimistic we do the same thing. It is necessary for the function to be parallel because it is not logic that these three functions cut themselves. Fig 1

For example:

Customer satisfaction on green supply products:

We have some methods that can be used for these criteria to satisfy the customer requirements, each of method can be doing some good in the period of time, and in the other period it can’t be a good method.

In the fact, this function in each time represents fuzzy triangular number, which is considering all three possibilities. Mahmoodi & Arshadi (2014)[14]

Why using FTF?

In many organizations some methods or some equipment could be pass the cycle life or the new equipment or method are better than the older ones. With FTF we could consider these things to our decisions and we could use the data of today for tomorrow and upgrade data if it was necessary. In fact the first decision can be a principle of the future decisions. Mahmoodi & Arshadi (2014)[15]
2.5. Proposed approach

This study attempts to apply fuzzy set theory and TODIM for ranking the risk of eight alternatives with fuzzy FMEA approach. The goal is to analyze how proposed method can be used to determine the risk factors of FMEA against alternative.

Step 1. A group of decision makers identifies the failure modes.

Step 2. Interpret the linguistic preference for the interval-valued TFNs. Use linguistic preference to convert the interval-valued TFNs into crisp value then perform fuzzy assessments according the Eqs (3)-(6) to remove the fuzziness and to aggregate the measures into a crisp value \( W_j \).

Step 3. Use Eq. (7) to determine the initial interval-valued TFNs decision matrix \( A \). The term \( \phi_c = (A_i, A_j) \) represents the contribution of the criterion \( c \) to the function \( \delta (A_i, A_j) \) when comparing alternative I with alternative j using Eqs. (10) and (11). The final matrix of dominance is obtained by summing the partial matrices of dominance for each criterion.

Step 4. Using the FTF for evaluating risk in different times, helped us to find out about the future failure of our system. We can do this in two ways. One is evaluating risk by collecting data from expert’s and their opinion about how will be the change of each criteria in the future or by using one function for each alternative to represent all effects of criteria.

Step 5. Calculate the global value of the alternatives by normalizing the final matrix dominance using Eq. (12). Ordering the values \( \xi_i \) provides the rank of each alternative. Important alternatives are those that have the highest \( \xi_i \). The function \( \phi_c = (A_i, A_j) \) permits the data to be adjusted to prospect theory’s value function. Fig (3).

Step 6. Combining the results of step 4 and step 5 to reach final table which represent weight of alternatives and criteria in different times.

Step 7. Using fuzzy time function for period of time needed in our research our managers need to know about the situation of criteria (FMEA factors) and alternatives in their organization.
3. AN ILLUSTRATIVE EXAMPLE

The proposed methodology is applied to manufacturing facility of a SME performing in an automotive industry. Major potential failure modes (PFMs) are identified by a group of experts in an assembly process at the manufacturing facility as non-conforming material (A), wrong die (B), wrong program (C). Excessive cycle time (D), wrong process (E), damaged goods (F), wrong part (G), and incorrect forms (H).

3.1. Results

This study evaluates the eight alternatives which have priority risk. First we must find out the weight of risk factors, and then find out about the alternative weights. With ranking the alternative the priority risk will be revealed. Expert opinion can has weight. For example some expert’s opinion might be more important than others. In this study we supposed all expert’s opinions are the same. we also research about the changing weight of criteria in the 10 and 20 months after this evaluation and will consider how much time this evaluation will be effective in our process with other hands we actually determine the life of alternatives.
Step 1. Table 3 presents the qualitative scales that require translation into interval-valued TFNs (see Table 2).

Step 2. Table 4 presents interval-valued TFNs, and the defuzzification process is employed in Eq. (7). The TFNs were applied to transform the total weighted matrices into interval performance matrices. The linguistic preferences were used to convert measures into a crisp value \( W_j \), Shown in Table 1. The TFNs were converted into crisp values shown in Table 6.

Step 3. The risk factors weight is calculated from Eq. (7) is shown in Table 5. Step 4. Using Eq. (9) to determine the initial interval valued TFNs decision matrix, the computations demonstrate how to determine the dominance measurements \( (A, B) \), which are presented. The computations represent the values of the measurements of dominance after the implementation of the mathematical formulation of the fuzzy TODIM method.

Step 5. Eq. (10) determines the values of the dominance measure, and \( \delta (A, B) \) is obtained for the different values of \( \phi (A, B) \) for all factors. The following computational processes are determined using Eq. (11). (Table 7)

Step 6. Eq (12) calculates the overall value of the alternatives by normalizing the corresponding dominance measurements. The rank of each alternative derives from ordering the alternative respective values. The global measures computed the complete rank ordering of all alternatives. In addition, a sensitivity analysis should then be applied to verify the stability of result based on the decision maker’s preferences. The results are presented in Table 8.

<table>
<thead>
<tr>
<th>Potential failure modes</th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Non-conforming material</td>
<td>F,F,MP</td>
<td>F,MG,MG</td>
<td>G,MG,G</td>
</tr>
<tr>
<td>(B) Wrong die</td>
<td>P,MP,MP</td>
<td>VNG,VG</td>
<td>MP,MP,P</td>
</tr>
<tr>
<td>(C) Wrong program</td>
<td>MP,MP,MP</td>
<td>VG,G,G</td>
<td>VP,MP,P</td>
</tr>
<tr>
<td>(D) Excessive cycle time</td>
<td>MP,MP,MP</td>
<td>F,MG,MG</td>
<td>G,MG,G</td>
</tr>
<tr>
<td>(E) Wrong process</td>
<td>F,F,MP</td>
<td>MG,MG,G</td>
<td>G,VG,G</td>
</tr>
<tr>
<td>(F) Damaged goods</td>
<td>MG,MG,MP</td>
<td>MG,G,M</td>
<td>MP,MP,F</td>
</tr>
<tr>
<td>(G) Wrong part</td>
<td>P,MP,VP</td>
<td>VG,VG,VP</td>
<td>VP,MP,P</td>
</tr>
<tr>
<td>(H) Incorrect forms</td>
<td>VP,VP,VP</td>
<td>VP,VP,VG</td>
<td>VP,VP,VP</td>
</tr>
</tbody>
</table>

Table 3. Evaluation of experts in linguistic variable for risk factor against PFMS.
### Table 4. Interval-valued TFNS decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1.66, 2.83), 4.33, (5.83, 6.83)</td>
<td>(3.83, 4.83), 6.33, (7.5, 8.33)</td>
<td>(5.66, 6.83), 8.33, (9.9, 9.33)</td>
</tr>
<tr>
<td>B</td>
<td>(0.55, 1.94), 3.44, (4.94, 5.94)</td>
<td>(7.5, 8.33), 9.66, (9.83, 10)</td>
<td>(3.03, 4.25), 5.54, (6.67, 7.49)</td>
</tr>
<tr>
<td>C</td>
<td>(0.55, 1.94), 3.44, (4.94, 5.94)</td>
<td>(6.5, 8.16), 9.33, (9.66, 10)</td>
<td>(2.03, 4.18), 5.39, (6.43, 7.11)</td>
</tr>
<tr>
<td>D</td>
<td>(0.83, 2.16), 3.66, (5.16, 6.16)</td>
<td>(3.83, 4.83), 6.33, (7.5, 8.33)</td>
<td>(5.16, 6.83), 5.33, (9.9, 9.83)</td>
</tr>
<tr>
<td>E</td>
<td>(1.66, 2.88), 4.33, (5.83, 6.83)</td>
<td>(4.83, 6.16), 7.66, (8.3, 9.66)</td>
<td>(6.5, 8.16), 9.33, (9.6, 10)</td>
</tr>
<tr>
<td>F</td>
<td>(4.83, 6.16), 7.66, (8.5, 9.66)</td>
<td>(4.83, 6.16), 7.66, (8.5, 9.66)</td>
<td>(0.83, 2.16), 3.66, (5.16, 6.16)</td>
</tr>
<tr>
<td>G</td>
<td>(1.26, 2.83), 3.73, (5.09, 6.04)</td>
<td>(8.5, 9.5), 10, (10, 10)</td>
<td>(3.17, 4.37), 5.6, (6.62, 7.46)</td>
</tr>
<tr>
<td>H</td>
<td>(1.26, 2.83), 3.73, (5.09, 6.04)</td>
<td>[0, 0], 0, (1.15)</td>
<td>[0, 0], 0, (1.15)</td>
</tr>
</tbody>
</table>

### Table 5. Factors (RPN) weight

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.39</td>
<td>0.25</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table 6. Matrix of Alternative score respect FMEA factors

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.41</td>
<td>0.36</td>
<td>0.23</td>
</tr>
<tr>
<td>B</td>
<td>0.49</td>
<td>0.095</td>
<td>0.41</td>
</tr>
<tr>
<td>C</td>
<td>0.46</td>
<td>0.13</td>
<td>0.4</td>
</tr>
<tr>
<td>D</td>
<td>0.45</td>
<td>0.34</td>
<td>0.2</td>
</tr>
<tr>
<td>E</td>
<td>0.49</td>
<td>0.32</td>
<td>0.17</td>
</tr>
<tr>
<td>F</td>
<td>0.22</td>
<td>0.24</td>
<td>0.53</td>
</tr>
<tr>
<td>G</td>
<td>0.54</td>
<td>0.06</td>
<td>0.4</td>
</tr>
<tr>
<td>H</td>
<td>0.26</td>
<td>0.37</td>
<td>0.37</td>
</tr>
</tbody>
</table>

### Table 7. Following computational processes Eq(11)

<table>
<thead>
<tr>
<th>δ</th>
<th>S</th>
<th>O</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A,B</td>
<td>-0.477</td>
<td>0.256</td>
<td>-0.713</td>
</tr>
<tr>
<td>A,C</td>
<td>-0.35</td>
<td>0.24</td>
<td>-0.699</td>
</tr>
<tr>
<td>A,D</td>
<td>-0.1</td>
<td>0.07</td>
<td>0.1</td>
</tr>
<tr>
<td>A,E</td>
<td>-0.447</td>
<td>0.1</td>
<td>0.144</td>
</tr>
<tr>
<td>A,F</td>
<td>0.26</td>
<td>0.17</td>
<td>-0.92</td>
</tr>
<tr>
<td>A,G</td>
<td>-0.573</td>
<td>0.279</td>
<td>-0.692</td>
</tr>
<tr>
<td>A,H</td>
<td>0.242</td>
<td>-0.197</td>
<td>-0.628</td>
</tr>
</tbody>
</table>
\[
\tilde{\xi} = \frac{\sum_{j=1}^{n} \delta(i, j) - \min \sum_{j=1}^{n} \delta(i, j)}{\max \sum_{j=1}^{n} \delta(i, j) - \min \sum_{j=1}^{n} \delta(i, j)} = \frac{-4.025 + 0.994}{0.07 + 0.994} = -3.031
\]

### Table 8. Final weights of Alternatives

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Xi)</td>
<td>-3.031</td>
<td>-1.69</td>
<td>-1.43</td>
<td>-1.48</td>
<td>-7.68</td>
<td>-5.19</td>
<td>-3.48</td>
</tr>
<tr>
<td>(G.W)</td>
<td>0.11</td>
<td>0.061</td>
<td>0.051</td>
<td>0.053</td>
<td>0.27</td>
<td>0.188</td>
<td>0.126</td>
</tr>
<tr>
<td>(R)</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

\(G.W=\text{Global weight}, R=\text{Ranking}\)

Step 7. The table 8 represents the weight of each criteria at present time but If we want to know in 10 or 20 month later which of this criteria have more RPN in our system what should we do? In here we use FTF for each of these 8 criteria. For example the company policy after this evaluation is reduce risk of first three criteria with weight of 0.3 and for others will be divided (0.1/5) but it’s not all the way through because we must consider time effect to this alternative and how they will be change. For this kind of things we use fuzzy time function which helps us to consider all situations it might happen in future.

For example we do this for criteria Numbers in table 9 are represent the severity of failure modes in 5, 10, 15,Month.in last column it’s empty and it’s means in which time this criteria will be removed from our system. 0.1t is function represent the normal increasing of this criteria during time which collected from experts. After 50 month of this research in organization, if with activities goes with this rate the failure E will self remove from the system. In this example we just suppose it’s 0.1t.

\[
y_e(t) = \begin{cases} 
0.1t - 0.3t + 7.68 & 0 \leq t < 15 \\
0.1t - 0.3t + 0.5t + 7.68 & t = 15, \\
0.1t - 0.3t + 7.18 & 15 < t < \infty 
\end{cases}
\]

\(t = \text{per month}\)
Table 9. The rate of RPN in of alternative E.

<table>
<thead>
<tr>
<th>Time (months)</th>
<th>RPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6.68</td>
</tr>
<tr>
<td>10</td>
<td>5.68</td>
</tr>
<tr>
<td>15</td>
<td>7.18</td>
</tr>
<tr>
<td>20</td>
<td>6.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (months)</th>
<th>RPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5.9</td>
</tr>
<tr>
<td>30</td>
<td>4.9</td>
</tr>
<tr>
<td>35</td>
<td>3.9</td>
</tr>
<tr>
<td>40</td>
<td>2.9</td>
</tr>
</tbody>
</table>

This numbers means in 15 month after this evaluation with policy of company will be reduced but in 20 month because of coming new technology to system this criteria will be increase and the cause of that it’s the cost of be familiar to new technology. And with this sequence after 10 month will be reduced to poor and this sequence will help us to forecast other periods and ending time.

\[ y_e(t) = 0.1t - 0.3t + 7.18 \quad 15 < t < \infty \Rightarrow t = 35.9 \Rightarrow 35.9 + 15 = 50.9 \]

Finally we can find out this criteria when will be removed effectively. This can be done for every criterion depend on time.

4. CONCLUSION

FMEA, designed to provide information for risk management decision–making, is a widely used engineering technique in industries. With TODIM approach decision makers can find out about any loss and gain between alternatives. For example in Table 7, we can find out the alternative A against B have -0.447 in factor severity, this means the severity of A is more critical than severity of B, but the in occurrence, occurrence of B is more critical than B. This represent a great view of system for decision makers. In TODIM, it’s easier to find out about the relationship between alternatives in each factor, but in the other solution like Fuzzy TOPSIS we can’t say the same thing.

The other aspect of this study is using FTF in the way of forecasting the future of the criteria which is very important for each firm to know when and where they most spends them money, which this work is done in supplier selection in green supply chain management. It will be suggested using fuzzy time function in other studies for predicting efficiency of system.

REFERENCES


