Analysis and Global Chaos Control of the Hyperchaotic Li System via Sliding Control

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Abstract

In this paper, we derive new results for global chaos control of chaotic systems via sliding control. We also explore the analysis and global chaos control of the hyperchaotic Li system (2005) using sliding control. The global chaos control results via sliding control have been established using Lyapunov stability theory. Numerical simulations using MATLAB are shown to validate and depict the effectiveness of the global chaos control of the hyperchaotic Li system.

Keywords

Sliding Control, Chaos, Hyperchaos, Control, Hyperchaotic Li System.

1. Introduction

Hyperchaotic systems have received considerable attention from many physical scientists and engineers. A hyperchaotic dynamical system is defined as a chaotic system having more than one positive Lyapunov exponent (LE) in its Lyapunov spectrum. The first hyperchaotic system was discovered by Rössler in 1979 [1].

During the last two decades, hyperchaotic systems have been studied using mathematical control theory and implemented using electronic oscillators [2-3]. Realization of hyperchaotic systems by circuit design has been applied in engineering areas such as secure communication [4-7], synchronization [8-9], encryption [10], etc. Thus, designing hyperchaos and control of hyperchaotic systems have become important research problems.

The control of chaotic system is to design state feedback control laws that stabilize the chaotic systems around the unstable equilibrium points. Chaos and control of chaotic dynamical systems are research problems that have both received rapid attention in the recent decades [11-20].

In this paper, we derive new results based on the sliding mode control [21-23] for the global control of hyperchaotic Li system ([24], 2005). In robust control theory, the sliding mode control method is often adopted due to its inherent advantages of easy realization, fast response and good transient performance and its insensitivity to parameter uncertainties and external disturbances.
This paper has been organized as follows. In Section 2, we describe the global control of a chaotic system using sliding mode control (SMC). In Section 3, we derive results for the global chaos control of the hyperchaotic Li system (2005). Section 4 contains a summary of the main results.

2. **GLOBAL CONTROL OF A CHAOTIC SYSTEM USING SMC**

In this paper, we consider a general chaotic system described by

\[ \dot{x} = Ax + f(x) + u \]  

In Eq. (1), \( x \in \mathbb{R}^n \) is the state of the system, \( A \) is the \( n \times n \) constant matrix of the system parameters, \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is the nonlinear part of the system and \( u \) is the control input.

The goal of the global chaos control problem is to seek a controller \( u \) such that

\[ \lim_{t \to \infty} \|x(t)\| = 0 \quad \text{for all } x(0) \in \mathbb{R}^n. \]  

To solve this global problem, we adopt the sliding control method.

As a first step, we define the control \( u \) as

\[ u = -f(x) + Bv \]  

where \( B \) is a constant \( n \times 1 \) matrix selected such that \( (A, B) \) is completely controllable.

Substituting (3) into (1), the state dynamics becomes

\[ \dot{x} = Ax + Bv \]  

which is a linear time-invariant control system having single input \( v \).

Thus, the original global chaos control problem is equivalent to the problem of stabilizing the zero solution \( x = 0 \) of the linear system (4) by means of a suitable choice of the sliding control.

In the following, we use Lyapunov stability theory to solve the equivalent control problem.

In the sliding control, we first define the variable

\[ s(x) = Cx = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \]  

where

\[ C = [c_1, c_2, \cdots, c_n] \]

is a constant vector to be determined.

In the sliding control, we restrict the motion of the system (4) to the sliding manifold defined by

\[ S = \{ x \in \mathbb{R}^n | s(x) = 0 \} \]

which is required to be invariant under the flow of the dynamics (4).
When in sliding manifold $S$, the system (4) satisfies the following conditions:

$$s(x) = 0$$

which is the defining equation for the manifold $S$ and

$$\dot{s}(x) = 0$$

which is the necessary condition for the state trajectory $x(t)$ of (4) to stay on the sliding manifold $S$.

Using (4) and (5), the equation (7) can be rewritten as

$$\dot{s}(x) = C [Ax + Bv] = 0$$

Solving (8) for $v$, we obtain the equivalent control law

$$v_{eq}(t) = -(CB)^{-1}CAx(t)$$

where $C$ is chosen such that $CB \neq 0$.

Substitution of (9) into the state dynamics (4) yields the closed-loop dynamics as

$$\dot{x} = \left[ I - B(CB)^{-1}C \right]Ax$$

The row vector $C$ is chosen in such a way that the system matrix of the controlled dynamics $\left[ I - B(CB)^{-1}C \right]A$ is Hurwitz, i.e. it has all eigenvalues in the open left-half of the complex plane. Then the controlled system (10) is globally asymptotically stable.

To design the sliding mode controller for (4), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q \text{sgn}(s) - k \ s$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0$, $k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (8) and (11), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1} \left[ C(kI + A)x + q \text{sgn}(s) \right]$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1} [C(kI + A)x + q], & \text{if } s(x) > 0 \\ -(CB)^{-1} [C(kI + A)x - q], & \text{if } s(x) < 0 \end{cases}$$
Theorem 1. The global control problem for the chaotic system (1) is solved by applying the feedback control law

\[ u(t) = -f(x) + Bv(t) \quad (14) \]

where \( v(t) \) is defined by (12) and \( B \) is a column vector such that \( (A, B) \) is controllable.

Proof. First, we note that substituting (14) and (12) into the chaotic dynamics (1), we arrive at the closed-loop error dynamics

\[ \dot{x} = Ax - B(CB)^{-1} \left[ C(kI + A)x + q\text{sgn}(s) \right] \quad (15) \]

To prove that the closed-loop state dynamics (15) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

\[ V(x) = \frac{1}{2} s^2(x) \quad (16) \]

which is a positive definite function on \( \mathbb{R}^n \).

Differentiating \( V \) along the trajectories of (15) or the equivalent dynamics (11), we get

\[ \dot{V}(e) = s(x)\dot{s}(x) = -ks^2 - q\text{sgn}(s)s \quad (17) \]

which is a negative definite function on \( \mathbb{R}^n \).

Thus, we have shown that \( V \) is a globally defined, positive definite, Lyapunov function for the state dynamics (15) and that \( \dot{V} \) is a globally defined, negative definite function.

Hence, by Lyapunov stability theory [25], it follows that the state dynamics (15) is globally asymptotically stable.

This completes the proof. \( \blacksquare \)

3. **Global Chaos Control of the Hyperchaotic Li System via Sliding Control**

3.1 Theoretical Results

In this section, we apply the sliding control results derived in Section 2 for the global chaos control of the hyperchaotic Li system (2005).

Thus, we consider the controlled hyperchaotic Li dynamics
\[
\begin{align*}
\dot{x}_1 &= a(x_2 - x_1) + x_4 + u_1 \\
\dot{x}_2 &= dx_1 - x_1x_3 + cx_2 + u_2 \\
\dot{x}_3 &= -bx_3 + x_4x_2 + u_3 \\
\dot{x}_4 &= x_2x_3 + rx_4 + u_4
\end{align*}
\]

(18)

where \( x_1, x_2, x_3, x_4 \) are state variables, \( a, b, c, d, r \) are positive, constant parameters of the system and \( u_1, u_2, u_3, u_4 \) are the controls to be designed.

We write the state dynamics (18) in the matrix notation as

\[
\dot{x} = Ax + f(x) + u
\]

(19)

Where

\[
A = \begin{bmatrix}
-a & a & 0 & 1 \\
d & c & 0 & 0 \\
0 & 0 & -b & 0 \\
0 & 0 & 0 & r
\end{bmatrix}, \quad f(x) = \begin{bmatrix}
0 \\
-x_1x_3 \\
x_1x_2 \\
x_2x_3
\end{bmatrix}
\]

and

\[
u = \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix}
\]

(20)

The 4-D system exhibits hyperchaotic behaviour when the parameter values are taken as

\[a = 35, \quad b = 3, \quad c = 12, \quad d = 7, \quad r = 0.6\]

(see Figure 1).

Figure 1. Phase Portrait of the Hyperchaotic Li System
Using the results of Section 2, we build a sliding controller for the global chaos control of the hyperchaotic Li system (18).

First, we define the control \( u \) as

\[
  u = -f(x) + Bv
\]

where \( B \) is carefully selected such that \((A,B)\) is completely controllable. We take \( B \) as

\[
  B = \begin{bmatrix}
    1 \\
    1 \\
    1 \\
    1
  \end{bmatrix}
\]

The sliding mode variable is selected as

\[
  s = Cx = \begin{bmatrix}
    1 & 1 & -3
  \end{bmatrix} x = 4x_1 + x_2 + x_3 - 3x_4
\]

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as \( k = 5 \) and \( q = 0.1 \).

From Eq. (12), we can obtain \( v(t) \) as

\[
  v(t) = 37.67 x_1 - 52.33 x_2 - 0.67 x_3 + 4.27 x_4 - 0.03 \text{sgn}(s)
\]

Thus, the required sliding mode controller is obtained as

\[
  u = -f(x) + Bv
\]

By Theorem 1, we obtain the following result.

**Theorem 2.** The hyperchaotic Li system (18) is globally asymptotically stabilized for all initial conditions with the sliding mode controller \( u \) defined by (25).

### 3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step \( h = 10^{-8} \) is used to solve the controlled hyperchaotic Li system (18) with the sliding mode controller \( u \) given by (25) using MATLAB.

In the hyperchaotic case, the parameter values are given by

\[
  a = 35, \quad b = 3, \quad c = 12, \quad d = 7, \quad r = 0.6
\]

The sliding mode gains are chosen as

\[
  k = 5 \quad \text{and} \quad q = 0.1.
\]
The initial values of the hyperchaotic Li system (18) are taken as

\[ x_1(0) = 20, \quad x_2(0) = -15, \quad x_3(0) = 8, \quad x_4(0) = 12 \]

Figure 2 illustrates the chaos control of the hyperchaotic Li system (18).

![Figure 2. Chaos Control of the Hyperchaotic Li System](image)

4. CONCLUSIONS

In this paper, we have derived new results for the global chaos control of chaotic systems via sliding control. We have also explored the analysis and global chaos control of the hyperchaotic Li system (2005). Our global chaos control results have been established using Lyapunov stability theory. Numerical simulations using MATLAB are also shown to illustrate the effectiveness of the SMC-based control results derived for the hyperchaotic Li system.

REFERENCES


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