MATHEMATICAL MODELING OF MAHESHWAR HYDRO-ELECTRIC PLANT AT NARMADA RIVER

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ABSTRACT

Economic hydropower plant generation scheduling is an important feature from the utility point of view, this scheduling is more important in that case when the plants are at the same river stream & owned by the different utilities. Various Conventional & Artificial intelligence methods have been used earlier for the hydro generation scheduling reported through researches and more, all are of them required the mathematical modeling of each & every hydro power plant.

Theoretical account of problem solving in general and especially methodology to develop the mathematical models of the hydroelectric power plants has been described in this paper. Reservoir elevation model, tail race elevation model and hydro turbine model have been developed for Rani Avanti Bai Sagar river bed hydroelectric power plants by collating actual plant data from competent authorities. Modeling of canal head hydroelectric power plants have been also included in this paper. This hydro power project is a part of cascade scheme at Narmada River in Madhya Pradesh, India.

KEYWORDS

Hydro-electric system, hydro turbine model, reservoir model, tailrace model, water flow equation

NOMENCLATURE

X_j^t	Reservoir storage of the j^{th} plant at time t	S_j^t	Spillage from the j^{th} river bed hydro power plant at time t
X_j^{min}	Minimum storage in j^{th} reservoir	Q_j^t	Total Discharge through the plant at time t including spillway and turbines discharge
X_j^{max}	Maximum storage in j^{th} reservoir		
H_j^t	Head for the j^{th} river bed hydro power plant at time t	IR_j^t	Irrigation requirement from j^{th} hydroelectric power plant at time t
H_{chj}^t	Head for the j^{th} canal head power plant at time t	EL_j^t	Evaporation loss in j^{th} hydroelectric power plant at time t
HF_j^t	Reservoir elevation of j^{th} plant at time t	ξ	Time delay for water flows from upstream plant to downstream plant
HT_j^t	Tailrace elevation of j^{th} plant at time t considering the total discharge	A_i	Hydro turbine model constants of river bed hydroelectric power plants
Y_j^t	Natural inflow in the j^{th} plant at time t	B_i	Reservoir elevation model constants of river bed and canal head power plants
U_j^t	Discharge through turbine of j^{th} plant at time t		
U_j^{min}	Minimum discharge through turbine of j^{th} plant	C_i	Tail race elevation model constants of River bed and canal head hydroelectric power plants
U_j^{max}	Maximum discharge through turbine of j^{th} plant	C_{chi}	Hydro turbine model constants of canal head power houses.

up	Index for immediate upstream plant
t, T	Time index, Total scheduled horizon
n	Total number of river bed & canal head hydroelectric power plants
j	Index of hydroelectric power plants
A_1 A_2 A_3 A_4 A_5 A_6	Hydro turbine model constant for river bed power house
B_1 B_2 B_3 B_4 B_5	Reservoir storage model constant
C_1 C_2 $C_3 C_4$	Tail race elevation model
QP	Quadratic Polynomial
CP	Cubical Polynomial
FP	Fourth order polynomial
FIP	Fifth order polynomial
SP	Sixth order polynomial
SEP	Seventh order polynomial
DS	Data set
RMSE	Root Mean Square Error
SSE	Sum of Square Error

1. INTRODUCTION

The problem of hydroelectric scheduling of Narmada Cascaded Hydroelectric System (NCHES) requires judicious mathematical modeling of the hydroelectric power plants. These mathematical models can be based primarily on empirical tables derived from field measurements or detail project reports of the concerned hydro power plant. In this research work information obtained from detail project reports have been used to develop the suitable mathematical model of the concerned hydro power plants. This paper discussed the type of data requirement; approach and development of mathematical models of Maheshwar Sagar river bed and canal head hydroelectric power plants.

In a scenario where management structures are different and in conflict with each other in their approach to maximize power generation, discharge and spillage become issues of contention. This is where the mathematical model required to explain the proper scheduling to help maximize the power generation.

Modeling of the hydro electric plant (HEP) is a very complex task and as such there is no uniform modeling as each one is unique to its location and requirement. The diversity of these designs makes its necessary to model each one individually. The parameters of modeling are nonlinear and highly dependent on the control variables. In each case of the power station study, the coefficients have to be determined by collating actual plant data.

1.1 Data Requirement

To develop the model of the hydroelectric power plant, the following data will be required & used while modeling:

- Hill Chart of the turbine unit of the plant
- Area capacity curve of the reservoir of the respective hydro-electric plant
- Tail race water rating curve of the hydro-electric plant
  - Total discharge at dam Axis
  - Discharge in Narmada river
- Water inflow into each reservoir as a function of upstream plant discharge, water transport delay and inflows between reservoirs.
- Water to energy conversion factor as a function of net head and discharge through each turbine.

1.2 Area of Modeling

Scheduling of hydro power plants needs an expression of output power in terms of head and discharges:

- Hydro-turbine Model
- Reservoir Elevation Model
- Tailrace Elevation Model

Above mentioned models for all RBPH's (River Bed Power House) & CHPH's (Canal Head Power House) of Narmada Canal Hydro Electric System (NCHES) will be developed using regression analysis by collating actual plant data:

- Turbine Efficiency Curve
- Area Capacity Curve
- Tailrace Water Level Curve

Other data requirement will be:

- Hourly Load Demand
- Hourly Irrigation Requirement
- Average Hourly Evaporation Loss
- Average Hourly Natural Inflows

2. NARMADA CASCADED HYDRO ELECTRIC SYSTEM

NCHES is located on the interstate river Narmada in India. This system is characterized by a cascade flow network, with water transport delay between successive reservoirs and variable natural inflows. This system has five major hydropower projects, namely:

- Rani Avanti Bai Sagar (RABSP);
- Indira Sagar (ISP);
- Omkareshwar (OSP);
- Maheshwar (MSP), located in the state of Madhya Pradesh, India;
- Sardar Sarovar (SSP) terminal project in the state of Gujarat.

Each project is having River Bed Power House(RBPH) at the right bank of the river. RABSP, ISP, OSP & SSP are the multipurpose projects which are also fulfilling the irrigation requirement of the nearby zone. To harness the potential energy of released water for irrigation Canal head Power House(CHPH) have also been installed or under implementation at the irrigation canal of the concerned multipurpose projects. All projects are located on the main stream of the river, hence a hydraulic coupling exists amongst them as shown in Figure 2, especially between ISP, OSP and MSP. The tailrace level of the ISP is matched with the full reservoir level of OSP. And similarly between OSP and MSP. Scheduling of hydro power plants needs an expression output power in terms of head and discharges, i.e., hydro turbine model, reservoir elevation model & tailrace elevation model.

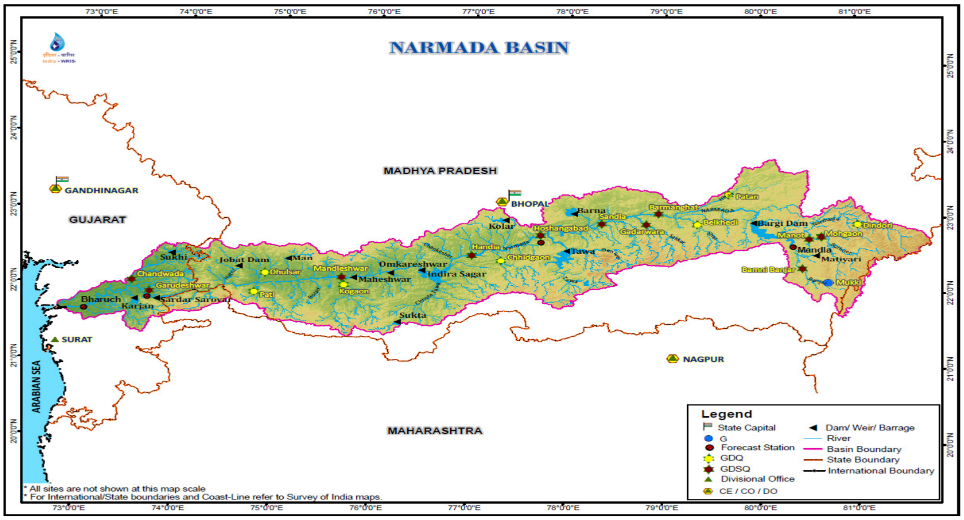

Figure 1. Hydro meteorological observation stations map

Above mentioned models for all RBPH's & CHPH's of NCHES have been developed using regression analysis by collating actual plant, i.e., turbine efficiency curve, area capacity curve & travelling time between successive reservoirs have been considered constant, i.e., 62 hours, 3hours, 17hours between Rani Avanti Bai Sagar to Indira Sagar, Indira Sagar to Omkareshwar, Omkareshwar to Maheshwar & Maheshwar to Sardar Sarovar hydro power plant respectively.

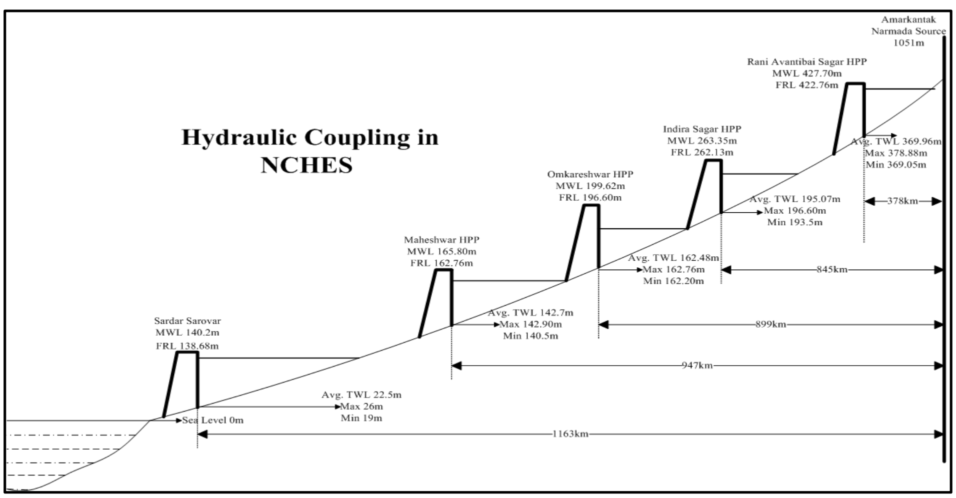

Figure 2. Hydro coupling in Narmada Cascaded Hydroelectric System

In a scenario where management structures are different and in conflict with each other in their approach to maximize power generation, discharge and spillage become issues of contention. This is where the mathematical model required to explain the proper scheduling to help maximize the power generation.

Modeling of the hydro electric plant is a very complex task and as such there is no uniform modeling as each one is unique to its location and requirement. The diversity of these designs makes its necessary to model each one individually. The parameters of modeling are nonlinear and highly dependent on the control variables. In case of the power station study, the coefficients have to be determined by collating actual plant data.

3. MAHESHWAR SAGAR HYDRO ELECTRIC PLANT MODEL

This is the last project across River Narmada to be constructed with on boundary of M.P. State. The project site is situated near Mandleshwar of Khargone district. This is a Hydro Power Project which has a power potential of 400 MW. Govt. of Madhya Pradesh has transferred this project to energy department in December-1088, and accordingly SM-PCL (Shree Maheshwar Power Company of Power Ltd.) is managing construction work of the project. Three units of project are almost completed.

Shree Maheshwar Hydel Power Corporation Ltd is the first private sector hydro project in the state of Madhya Pradesh. It is a 400 MW run-off-the-river project on the River Narmada, located at Mandleshwar, District Khargone, 108 Km. south-west of Indore. The Maheshwar Project will reduce the peak-power shortage in the state of Madhya Pradesh by over 25% as well as improve the socio economic condition of the region.

- The Maheshwar Hydel Power Project is located at Mandaleshwar, 108 km south-west of Indore (approx 2hr drive from Indore Airport), Madhya Pradesh, India
- It is the First Hydro Power Project to have been awarded to the Private Sector.
- The Maheshwar Hydel Project has the least submergence amongst all the other projects in the Narmada Valley
- Maheshwar is a run-of-the-river, peaking power project which will reduce the peaking power shortage of the state by about25%
- The Project is the only Independent Power Project in Madhya Pradesh with an escrow agreement executed with MPEB, the state electricity board, and also has a 35-year Power Purchase Agreement executed with MPEB.
- Will meet the drinking water requirement of Indore and the adjoining areas
- Is of great importance to the Nation and the State of Madhya Pradesh.

3.1 Power & Water

- The project will assure availability of water and electricity to the industrial areas of Indore and Dewas.
- The peaking power shortage of the state will reduce by almost 25%.
- Voltage fluctuation and the frequent power-shedding problem in the region will reduce.
- The water level of the adjoining areas will move up, saving a substantial amount of electricity in the State, and particularly in the four affected tehsil, for pumping water, for irrigation and drinking purposes.

Name of the Hydro Electric Project	Maheshwar Hydroelectric Project
Type of project	Major
Purpose of project	Hydroelectric
Name of the River	Narmada
Basin Name	Narmada
State Name	Madhya Pradesh
Inter Basin Project	No
Project Sharing	None
Owner Name	SMHPC/SKUMAR
Project Owner Type	Private
Total Installed Capacity (MW)	400

Table 1: Salient features of Maheshwar hydroelectric project

4. HYDRO TURBINE MODEL

It is a very important to model the hydro turbine as it is a part of objective function in the hydroelectric scheduling problem. This model relates the magnitude of power generated in terms of head and discharge. Head is a difference of reservoir elevation and tailrace elevation assuming head losses negligible. As discussed in previously that hydro turbine, reservoir elevation and tailrace elevation model can be developed using turbine efficiency curve, area capacity curve and tailrace water level curve. Hydro turbine model or hydro generator characteristic equation relates the amount of power generation with head and discharges through turbine. As in this case power is dependent variable whereas head and discharge are the independent variables, so it can be modeled using multiple regression analysis or curve fitting as discussed in subsequent section. To develop the mathematical models of the individual hydro plants least square method has been used.

$$Pj^t = A1(Hj^t)^2 + A2(Uj^t)^2 + A3(Hj^t)(Uj^t) + A4(Hj^t) + A5(Uj^t) + A6 \quad (1)$$

Where the constants of the hydro turbine models for river bed power houses are calculated as A1 = 0.0007, A2 = 0.0002, A3 = 0.009, A4 = -0.059, A5 = 0.001, A6 = -3.015. Also, in the statistical analysis the RMSE value is 0.368139 & Standard Deviation is 7.80867606.

5. RESERVOIR ELEVATION MODEL

Modeling of water stored in a reservoir forms a crucial part at any hydroelectric operational study as it determines the energy content and gross head of that plant. Reservoir elevation model can be developed using area capacity curve. This model relates the level of reservoir in terms of reservoir storage or volume of stored water in reservoir. Since in this case one is the independent variable and other is dependent variable. Hence this model can be obtained by simple regression analysis using MATLAB curve fitting toolbox. It is affected by the inflows to reservoir as well outflows from reservoir.

This model for hydro electric plants of Maheshwar Sagar hydroelectric power plant has been developed using data sets of area capacity curve. MATLAB curve fitting toolbox has been used to determine best suited model and corresponding correlation coefficients based on statistical analysis. Simulation results of reservoir elevation model is shown in Fig. 3 for river bed hydro electric power plant of Maheswar Sagar hydroelectric system.

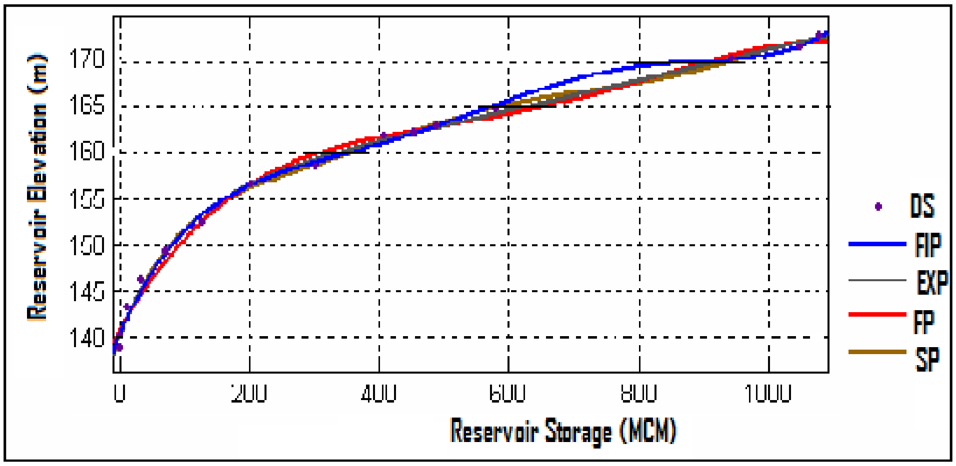

Figure 3(a)

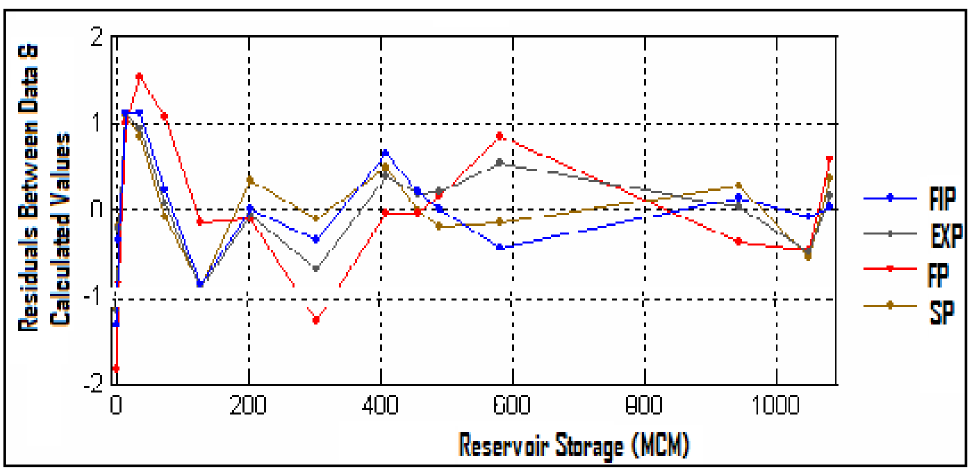

Figure 3(b)

Figure 3. Simulation results for reservoir elevation model of Maheshwar RBPH.

Figure 3 dealt with simulation result of reservoir elevation model for Maheshwar hydro electric power plant. Here data sets obtained from area capacity curve as given by concerned authority of Maheshwar tested for suitable fit using MATLAB curve fitting toolbox with readily available models i.e. fifth order polynomial FIP, fourth order polynomial FP, exponential EXP, sixth order polynomial SP as shown in Figure 3(a). Whereas its subpart Figure 3(b) indicates the residuals between data sets and calculated value obtained from fitted models. Best fitted model is selected according to the statistical analysis given in Table 2 and for this plant also exponential model is found to more suitable as it has favourable values of Root Mean Square Error and R square which are 0.7124 and 0.9968 respectively, which are the lowest in case of fifth degree polynomial. Hence a fifth order polynomial equation satisfying the characteristic of the area capacity curve and the constants of the equations are:'

$$HF_j^t = B_1 \exp(B_2 \times X_j^t) + B_3 \exp(B_4 \times X_j^t) \qquad (2)$$

B1=155.1, B2=9.681*10-5, B3=-14.87, B4= -0.01085

Table 2: Suitable model of reservoir elevation of Maheshwar River Bed hydro electric power plant.

Fitted model	Coefficients	SSE	RMSE	Adj R-square	R-Square
$HF_j^t = B_1 \exp(B_2 \times X_j^t) + B_3 \exp(B_4 \times$	B_1=155.1 B_2=9.681*10^{-5} B_3=-14.87 B_4=-0.01085	5.58	0.7124	0.995	0.9968
$HF_j^t = B_1(X_j^t)^4 + B_2(X_j^t)^3 + B_3(X_j^t)^2 + B_4(X_j^t) +$	B_1=-1.12*10^{-10} B_2=2.99*10^{-7} B_3=-0.00028 B_4=0.1225 B_5=140.9	11.58	1.07	0.99	0.9933
$HF_j^t = B_1(X_j^t)^5 + B_2(X_j^t)^4 + B_3(X_j^t)^3 + B_4(X$ $\ldots + B_5(X_j^t) + B_6$	B_1 = 3.6*10^{-13} B_2 = -1.08*10^{-9} B_3 = 1.18*10^{-6} B_4 = -0.00059 B_5 = 0.1597 B_6 = 140.4	5.91	0.81	0.994	0.996
$HF_j^t = B_1(X_j^t)^6 + B_2(X_j^t)^5 + B_3(X_j^t)^4 + B_4(X_j^t)^3 + B_5($ $\ldots B_6(X_j^t) + B_7$	B_1 = -7*10^{-16} B_2 = 2.5*10^{-12} B_3 = -3.49*10^{-9} B_4 = 2.41*10^{-6} B_5 = -0.00087 B_6 = 0.1807 B_7 = 140.2	4.953	0.786	0.995	0.997

6. TAILRACE ELEVATION MODEL

As per design of the hydroelectric plant, the station discharge either through turbine or spillway can raise the tailrace elevation, which decreases the effective head. Tail race elevation model can be developed using tailrace water level curve. This model related the water level in tail race channel with total discharge (discharge through turbine & spillway) through hydro power plant. Here also, as same reservoir elevation model one is the independent variable and other is dependent variable. Thus, the model can be obtained by simple regression analysis using MATLAB curve fitting toolbox.

Figure 4 shows the simulation results of Tailrace elevation model of Maheshwar hydroelectric power plant. Here data sets ( ) obtained from tail race water level curve as given by concerned authority of Maheshwar hydroelectric plant used for the development of mathematical model for tail race elevation. Various models like cubic polynomial, fourth degree polynomial, fifth degree polynomial, and exponential have been tested for suitable fit with data sets as given in Figure 4(a).Residual plot between data sets and its corresponding values obtained from the considered models are shown in Figure 4(b). Details of the fit are given in Table 3

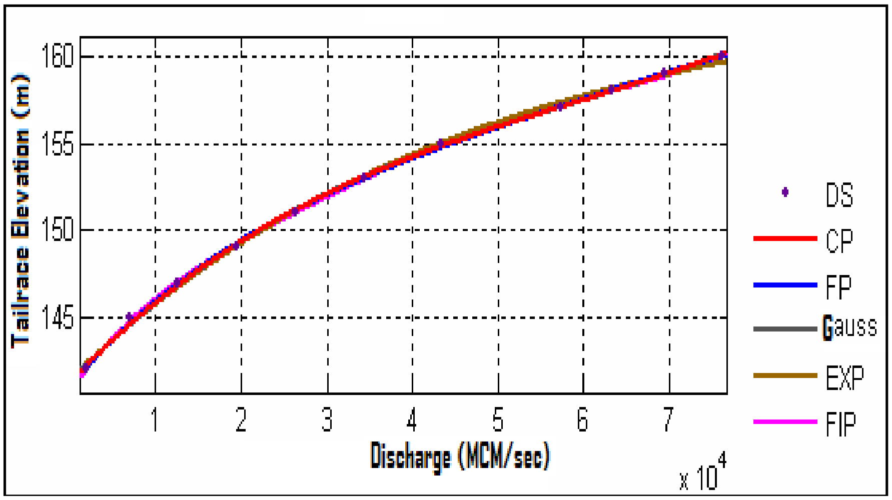

Figure 4(a)

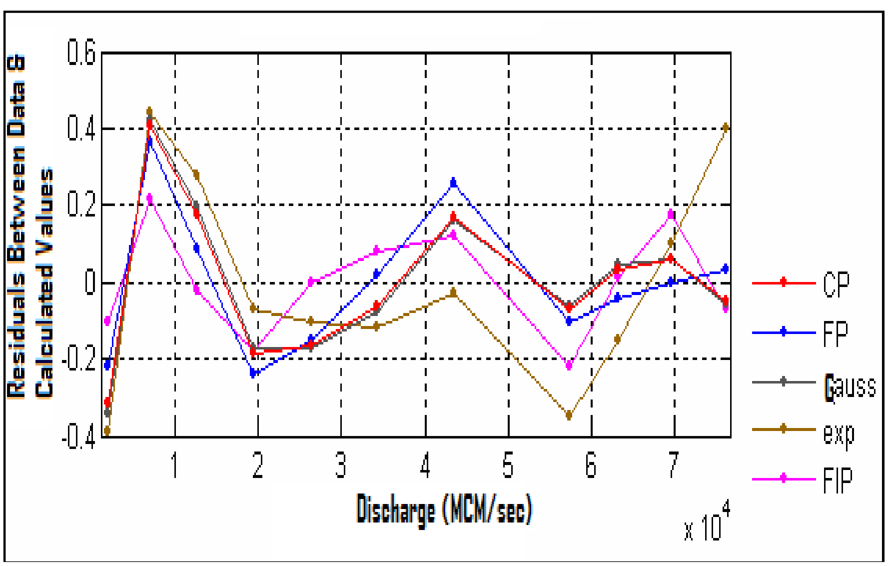

Figure 4(b)

Figure 4. Simulation results for tail race elevation model of Maheshwar RBPH.

$$HT_j^t = C_1(Q_j^t)^4 + C_2(Q_j^t)^3 + C_3(Q_j^t)^2 + C_4(Q_j^t) + C_5 \tag{3}$$

Coefficients of fitted model judged by its lowest RMSE & SSE values 0.3311 & 2.1927 respectively, are: C1= -5.42*106, C2=6.96*105, C3=-3.19*104, C4=853.6, C5=140.8. Hence fourth degree polynomial fit is suitable for the tailrace elevation modeling of Maheshwar Sagar Hydroelectric Power Plant.

Table 15: Suitable model of tail race elevation of Maheshwar River Bed hydro electric power plant.

Fitted model	**Coefficients**	**SSE**	**RMSE**	**Adj R-square**	**R-Square**
$HT_j^t = C_1(Q_j^t)^4 + C_2(Q_j^t)^3 + C_3(Q_j^t)^2 + C_4(Q_j^t) + C$	C_1= -5.42*10^6 C_2=6.96*10^5 C_3=-3.19*10^4 C_4=853.6 C_5=140.8	2.1927	0.3311	0.996	0.99725
$HT_j^t = C_1 \exp(C_2 \times Q_j^t) + C_3 \exp(C_4 \times Q_j^t)$	C_1=151 C_2=0.7858 C_3= -9.839 C_4= -55.99	4.446	0.4601	0.997	0.99444
$HT_j^t = C_1 \exp(-((Q_j^t - C_2)/C_3)^2)$	C_1=190 C_2=.2655 C_3=.4944	29.37	1.15	0.959	0.963
$HT_j^t = C_1(Q)^5 + C_2(Q)^4 + C_3(Q)^3 + C_4(Q)^2 + C_5(Q)$	C_1=1.86*10^8 C_2=-3.26*10^7 C_3=2.07*10^6 C_4=-5.95*10^4 C_5=1042 C_6=140.6	1.307	0.2623	0.997	0.9984
$HT_j^t = C_1(Q_j^t)^3 + C_2(Q_j^t)^2 + C_3(Q_j^t) + C_4$	C_1=7.18*10^2 C_2=.9813 C_3=613.3 C_4=141.2	5.469	0.51	0.992	0.993

6. WATER FLOW EQUATION

Hydro plants can be located either at different stream or same stream (series/cascaded) or multi chain type (combination of different stream and same stream). In cascaded system, discharge from immediate upstream reservoir is one of the factor in deciding the operating head & reservoir volume of the immediate downstream plant. Water flow equation relates the content of reservoir at any instant with due consideration of time delay to reach water from its upstream plant to the immediate downstream plant.

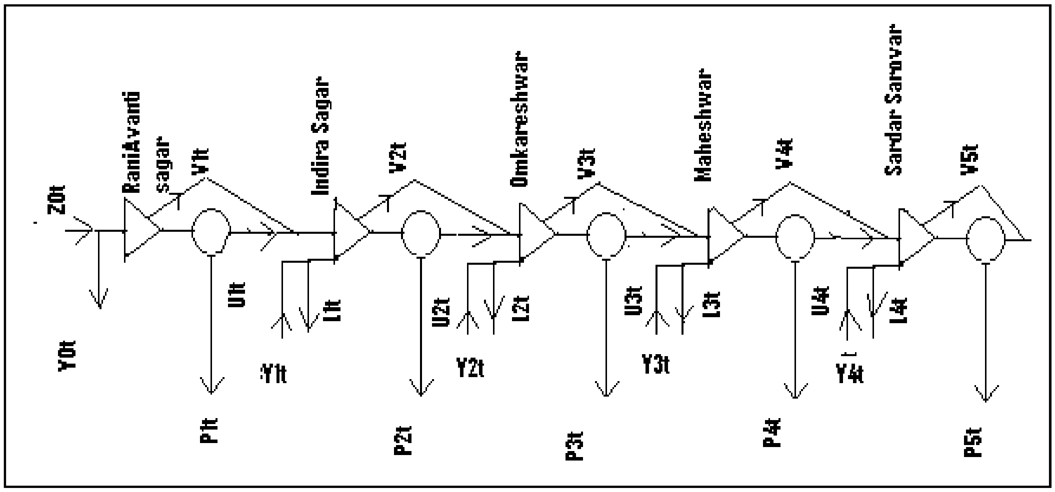

Figure 5. Water flow diagram in cascaded hydro electric system.

Total Inflow into station j at time t

$$I_j^t = Y_j^t + Q_{up}^{t-\xi} \qquad (4)$$

where ($Q_k^{t-\xi} = U_{up}^{t-\xi} + S_{up}^{t-\xi}$) (5)

$Q_{up}^{t-\xi}, U_{up}^{t-\xi}, S_{up}^{t-\xi}$ are total discharge, discharge through turbine, spillage which reaches to downstream plant after delay time .

Total outflow from station j at time t

$$O_j^t = U_j^t + S_j^t + EL_j^t + IR_j^t \qquad (6)$$

$$X_j^t = X_j^{t-1} + Y_j^t - U_j^t - S_j^t - EL_j^t - IR_j^t + (U_{up}^{t-\xi} + S_{up}^{t-\xi})$$

Storage in reservoir at time t

$$X_j^t = X_j^{t-1} + Y_j^t - U_j^t - S_j^t - EL_j^t - IR_j^t + (U_{up}^{t-\xi} + S_{up}^{t-\xi}) \qquad (7)$$

If $Y_j^t, EL_j^t, S_j^t, S_{up}^{t-\xi}, IR_j^t$ are assumed negligible hence the water continuity equation is

$$X_j^t = X_j^{t-1} - U_j^t + U_{up}^{t-\xi} \qquad (8)$$

In water continuity eq. natural inflows, evaporation loss and irrigation requirement for each hydro plant of NCHES are taken from reservoir operation Table of the concerned power plant.

Approximate delay time between consecutive reservoir i.e. Rani Avanti Bai Sagar to Indira sagar, Indira sagar to Omkareshwar, Omkareshwar to Maheshwar, Maheshwar to Sardar Sarovar are 43, 4, 3, 17 hrs respectively.

6. CONCLUSION

The suitable mathematical models for the river bed power house of Maheshwar Sagar hydroelectric system have been developed from the typical curves and data received from the detailed project reports of concerned power plant obtained by competent authorities. The model coefficients and a comparison of the actual data and the curves predicted using the model yield satisfactory results. These mathematical models are be used to develop an optimization problem formulation for daily optimal hourly generation scheduling of the Narmada Cascaded Hydroelectric System (NCHES).

REFERENCES

[1] A. Mahor & Dr. S. Rangnekar, “Mathematical Modeling of Omkareshwar Hydro-electric plant at Narmada River”, HYDRO NEPAL Issue No. 3, July 2008.

[2] A. Mahor & Dr. S. Rangnekar, “Least Square Approach to Model Indira Sagar Left Bank Canal Head Power House”, Renewable Energy and Environment for Sustainable Development, pp. 1086-1092.

[3] Naresh R. & Sharma J, “Short term hydro scheduling using Two phase neural network”, International Journal of Electrical Power and Energy Systems, Volume 24, issue 7, pp. 583-590, October 2002.

[4] S. Kumar's Ltd. (Shree Maheshwar Hydel Power Corporation Limited), April 1993, Updated Project Report ,Maheshwar Hydroelectric Power Project.

[5] Naresh R., Sharma J., “Short term hydro scheduling using ANN”, IEEE Transactions on Power Systems, vol. 15, no. 1, Feb. 2000.

[6] Christiano Lyra & Luiz Roberrto, “A Multiobjective Approach to the Short-term Scheduling of a Hydroelectric Power Sytem”, IEEE Transactions on Power Systems, Vol. 10, No. 4, November 1995.

[7] A. Mahor & N Kishore,“Optimal Generation Scheduling of Cascaded Hydroelectric System using Natural Exponential Inertia Weight PSO”, Global Journal of researches in engineering Electrical and electronics engineering, Volume 12 Issue 1, Version 1.0, January 2012

[8] Allen J. Wood, Bruce F. Wollenberg,“Power, Generation, Operation & Control” Wiley India (P) Ltd., 2nd Edition, 2013

[9] B.H.Khan,“Non-conventional Energy Resources” McGraw Hill Education,2nd Edition, 2013

[10] D.P.Kothari, K.C.Singal, Rakesh Ranjan, “Renewable Energy Sources & Emerging Technologies” PHI Pvt. Ltd.,2nd Edition, November,2011

[11] Jefferson W. Tester, Elisabeth M. Drake, “Sustainable Energy- Choosing Among Options” PHI Pvt. Ltd., MIT Press, 2005

[12] India WRIS Wiki http://india-wris.nrsc.gov.in/wrpinfo/index.php?title=Main_Page

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