CONSTITUTIVE MODELING FOR FLOW STRESS DETERMINATION USING DIMENSIONLESS PROPORTIONALITY RELATIONSHIP

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ABSTRACT

Constitutive modeling containing the relationship between stress, strain, strain rate and temperature is the essential input for understanding the thermo mechanical deformation processing results. In this research article, the dynamics of deformation was studied on the basis of a phenomenological mathematical model. Sustained tuning was earmarked by the dimensionless proportionality relationship as an additional assignment to modeling. Results show a new homeostatic steady stress and strain distribution pattern with respect to the applied strain rate & temperature. The results are compared with Johnson-Cook Model (JC) and found better than it. The driving spur is the intake of hot compression test by hiding the torsion test requirement, thereby creating an additional advantage over JC Model that requires both. It also begs a favor over N.S. Babu Model which requires 48 numbers of constants to predict the flow stress relationship which is 12 times more as compared to present approach.

KEYWORDS

Hot deformation, Constitutive model, Flow stress, Thermo-mechanical process, Johnson-Cook model.

1. INTRODUCTION

The mechanics of materials under thermal processing is generally characterized by constitutive modeling which relates the effective flow stress of deformation to the strain, strain rate, and temperature [1, 2]. The equations governing such mathematics are evaluated by a set of experimental outcomes in the deformation domain of the practical curricula. Hence, the explanation of the dynamic flow behavior of these metals and alloys under hot deformation circumstances is accompanied by defining the constitutive relationship correspond to the intrinsic changes due to microstructural upheavals like dynamic recrystallization and recovery in the bulk [3]. Many researchers have conducted studies on modeling the dynamic flow behavior of a work piece material for defining the process designing methods governing improve productivity at reduced cost of processing [4]. Such findings helps to detect and eliminate the experimental error caused due to sudden change in physical conditions like voltage fluctuation. The constitutive modeling helps us to predict the theoretical result at those circumstances where real experiments require huge potential and manhandling. The precision of the result depends on the efficiency of a model. For example, a case study made by Dipti et al. shows the competence of various models to predict the elevated temperature flow behavior of reactor structural material [5]. Thus, the modeling effect depends on the type of precision take under care to formulate the description of the phenomenon.

This paper reflects an innovative scheme for the determination of flow stress using dimensionless proportionality relationship. This strategy will allow us to input the unit in any SI or non SI dimension. The premier motive behind the present study on hot deformation practices of an alloy is to obtain a good workable condition to confine the particular alloy for specific mechanical application with respect to the applied regime of temperature, strain and strain rate. This type of computational modeling will also help us to depict the compact or individual effect of a workable parameter on others. The present model illustrates the relationship between different deformation parameters which is missed in many other conventional models. The limited inclusion of the constants further describes the worth of the present modeling. Comparisons of the results have been made in figures to illustrate the effectiveness of the present modeling. The model predictions are found to be in good agreement with published experimental findings. The driving stimulus is the consideration of only hot compression test as the only input for modeling a nonlinear elastic deformation without using the torsion test results. Thus, it creates goodness over JC Model that requires both types of inputs. It also limits the number of constants, so produces a forwarding mark over N.S. Babu Model [6] which needs 48 constants to define the constitutive aspect of deformation i.e. the number of constants are 12 times more as compared to present approach. The present model lessens two constants of Modified JC Model [1] & Series Expansion Method [2], thus add another fact for its favor.

2. MATERIAL MODELS

Many such works have previously been reported by many researchers. For example, the two basic power laws [7, 8].

$$\sigma = k\varepsilon^{n} \tag{1}$$

$$\sigma = k_1\dot{\varepsilon}^{m} \tag{2}$$

Where, σ is the equivalent stress; ε and $\dot{\varepsilon}_0$ are the equivalent plastic strain and strain rate respectively. k , k_1 are constant for particular strain, strain rate and temperature. Both these equations join only two parameters at a time and neglect the direct effect of other workable parameter.

On the next side, *Arrehenius power law, Exponential law, Hyperbolic sine law* were proposed. The equations concerning them are respectively shown below [9, 10, 11]

$$\dot{\varepsilon}\exp(Q/RT) = A\sigma^{n} \quad (3)$$

$$\dot{\varepsilon}\exp(Q/RT) = A`\exp(\beta\sigma) \quad (4)$$

$$\dot{\varepsilon}\exp(Q/RT) = A''(\sinh\alpha\sigma)^{n'} \quad (5)$$

where, A'', α, n' are constants and fixed for a system. There is the limitation of the applicability of the above equations (3) to (5) [12]. For example, equation (3) holds well for low level of stress, whereas equation (4) is suitable for high stress level. However, equation (5) performs fine at all amplitude of stress. All these equations, lacks one deformation workable parameters i.e. strain, thereby unable to include the net aspects of the outcome.

To complete the prospect of modeling and including all the parameters, fewer more models have been proposed. *Johnson-Cook Model* is among the most widely used model [13]. It connects all the deformation parameters in the following compact form.

$$\sigma = \left[A_1 + A_2\ \varepsilon^{n}\right]\left[1 + A_3 \ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right]\left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^{m}\right] \quad (6)$$

where, $\dot{\varepsilon}_0$ is a reference strain rate taken for normalization; A_1 is the yield stress and A_2 is the strain hardening factor, whereas A_3 is a dimensionless strain rate hardening coefficient. Parameters "*n* & *m*" are the power exponents of the effective strain and strain rate. Torsion test is more complicated and costlier than compression test. Johnson-Cook model requires the data from both torsion test and compression test.

The material model by *N. S. Babu et al* can also be used for modeling the flow stress [6]. It contains all the targeting parameters by assuming the following form,

$$\ln\sigma = \sum_{i=1}^{I}\sum_{j+1}^{J}\sum_{k=1}^{K} C_{ijk}\varepsilon^{i-1}\Theta^{j-1}(\ln\dot{\varepsilon})^{k-1} \quad (7)$$

where, Θ = 1000/T and T is taken in Kelvin. This formula includes the four parameters in *48* numbers of constants that requires a huge calculation steps to derive all the arbitrary values. The difficulty also arises during estimation of flow stress, since three values of dependent variable and *48* values of arbitrary constants has to be substituted for each theoretical calculation. The present model has 12 times lesser number of constants which provide us the opportunity to signify the physical meaning of constants in modeling. Not only this, the present model lessens two constants of *Modified JC Model* [1] & *Series expansion Method* [2], thus reduces the step of calculation and adds another fact to its benefit.

3. PROPOSED MODEL & ITS DEVELOPMENT

This model illustrates a dimensionless effect of parameters in defining the flow stress of the alloy. The Flow Stress (σ) in functional form is taken as the function of normalized independent variables,

$$\sigma = f\left(\varepsilon, \dot{\varepsilon}^*, T^*\right) \tag{8}$$

Where, ε represents the effective strain; $\dot{\varepsilon}^*, T^*$ are the normalized strain rate and normalized temperature. The equation representing this model is a simpler four constant relationship between all the flow parameters as observed below.

Let us initially assume the flow stress is expressible as a function of strain rate & temperature

$$\sigma = A\varepsilon\,\xi\left(\dot{\varepsilon}^*, T^*\right) \tag{9}$$

But, at $\varepsilon = \varepsilon_0$, we have

$$\sigma_0 = A\varepsilon_0\,\xi\left(\dot{\varepsilon}^*, T^*\right) \tag{10}$$

Eq(9) - Eq (10), yields

$$\sigma - \sigma_0 = A\left(\varepsilon - \varepsilon_0\right)\xi\left(\dot{\varepsilon}^*, T^*\right) \tag{11}$$

Or,

$$\sigma = \sigma_0 + A\left(\varepsilon - \varepsilon_0\right)\xi\left(\dot{\varepsilon}^*, T^*\right) \tag{12}$$

Due to the nature of consistency in the equation we can assume,

$$\kappa = \frac{\sigma_0 + A\left(-\varepsilon_0\right)\xi\left(\dot{\varepsilon}^*, T^*\right)}{\xi\left(\dot{\varepsilon}^*, T^*\right)} \tag{13}$$

Which gives the deforming flow stress in terms of $\xi\left(\dot{\varepsilon}^*, T^*\right)$. To elaborate this term, we further consider $\xi\left(\dot{\varepsilon}^*, T^*\right)$ in the span of $ln\ \dot{\varepsilon}^*$ & T^*. i.e.

$$\xi\left(\dot{\varepsilon}^*, T^*\right) = exp\left\{span\left(ln\ \dot{\varepsilon}^*, T^*\right)\right\} \tag{14}$$

Or, in particular,

$$\xi\left(\dot{\varepsilon}^{*},T^{*}\right)=exp\left\{mln\ \dot{\varepsilon}^{*}+cT^{*}\right\} \tag{15}$$

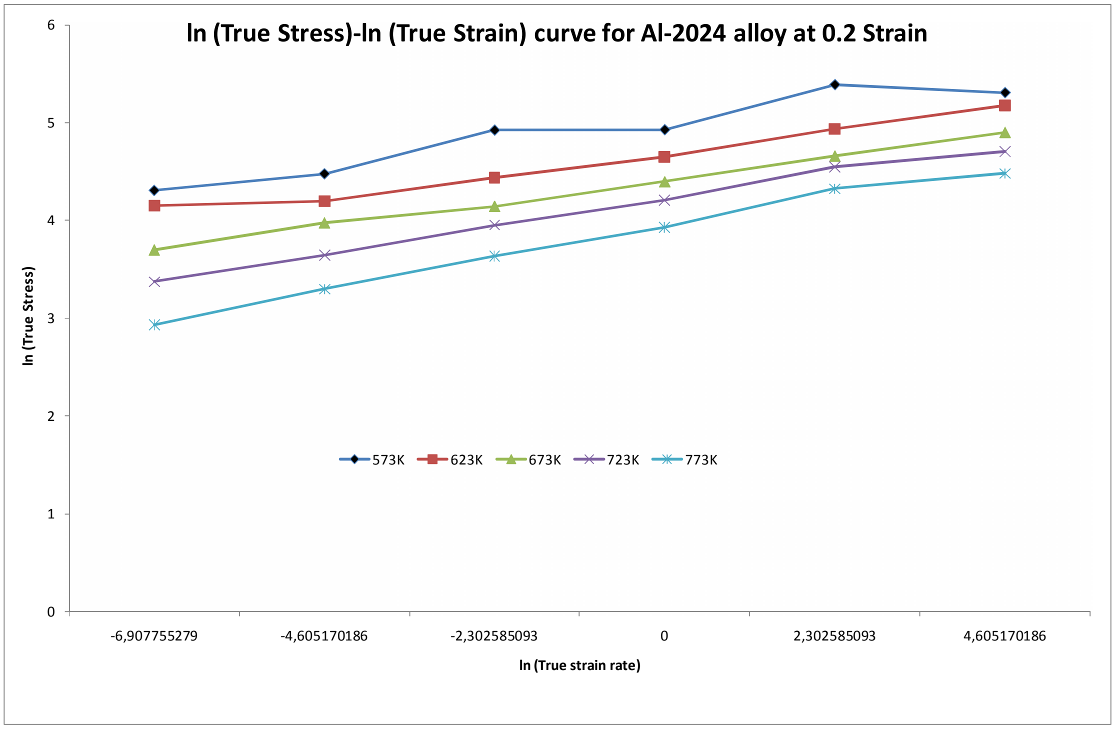
Where, the normalized terms are assumed to be, $\dot{\varepsilon}^{*}=\left(\dot{\varepsilon}/\dot{\varepsilon}_{0}\right)$, $T^{*}=\left(\mathrm{T}_{m}-\mathrm{T}\right)/\left(\mathrm{T}-\mathrm{T}_{r}\right)$. Here, T_{m} & T_{r} respectively represents the melting point and the room temperature as the reference temperature of the alloy, $\dot{\varepsilon}_{0}$ is taken as the reference strain rate. The alloy taken under study is Al-2024 Aluminum alloy. For establishing the relationship, *ln* (True stress)-*ln* (True strain rate) plots has been made at ε = 0.1 to 0.5. This will help to establish the σ-ε relationship by the power law relationship,

$$\sigma=k\dot{\varepsilon}^{m} \tag{16}$$

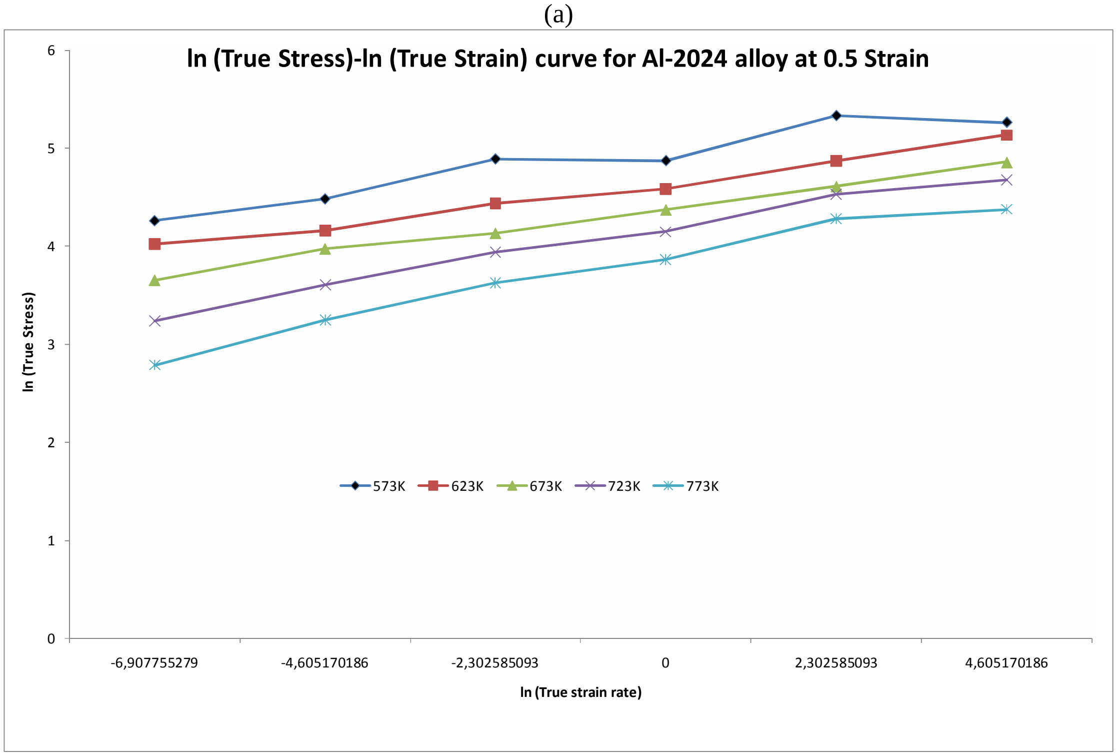
In this way, the *ln*(True stress)-*ln*(True strain rate) plots is shown in Fig-1 at ε = 0.2, 0.5. With the slopes obtained for 0.1, 0.2, 0.3, 0.4 and 0.5, various values of '*m*' were averaged out to finally obtain strain rate sensitive parameter "*m*" for the system. Similarly, other relation plots were made w.r.t other parameters to complete the relationship.

4. RESULT AND DISCUSSION

The values of constants used in obtaining the value of constants for Al 2024 alloy is shown in Table-1. A Comparison between the experimental results, the results from Johnson-Cook Model & calculated results by present model at different temperatures w.r.t. other parameters are made in terms of the flow stress curves in Fig-2. This model is also developed for other alloys and the results are summarized in Table-2. In all the cases $\dot{\varepsilon}_{0}$ is taken to be unity and T_r is allotted 298K.



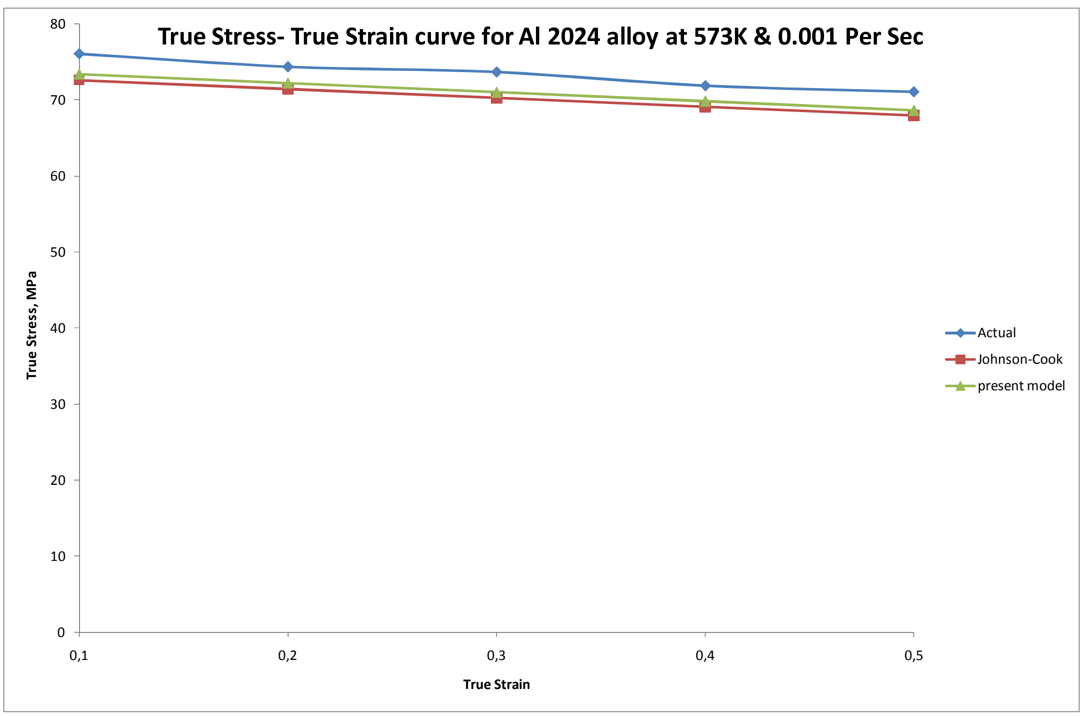
(a)



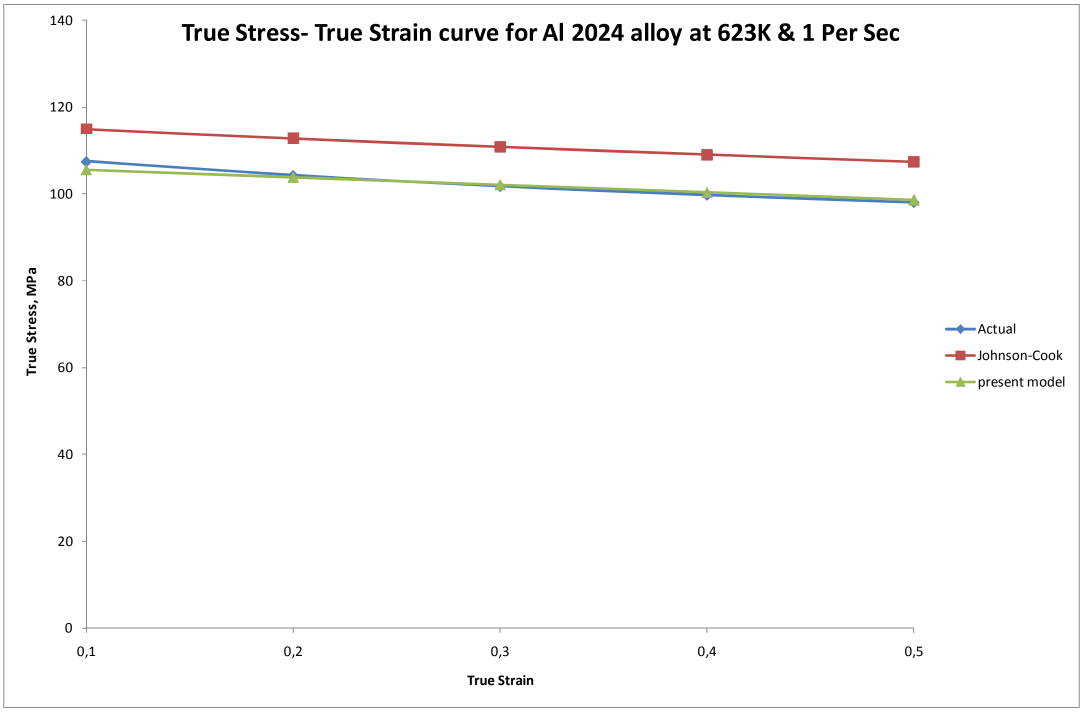
(b)Fig-1: σ-ε relationship at (a)$\varepsilon = 0.2$ (b) $\varepsilon = 0.5$

Constants & Parameters	***Values***
κ	38.518
A	– 6.160
m	0.110
C	1.1561
$\dot{\varepsilon}_0$	1 S^{-1}
T_m	911K
T_r	298K

TABLE 1. VALUE OF CONSTANTS



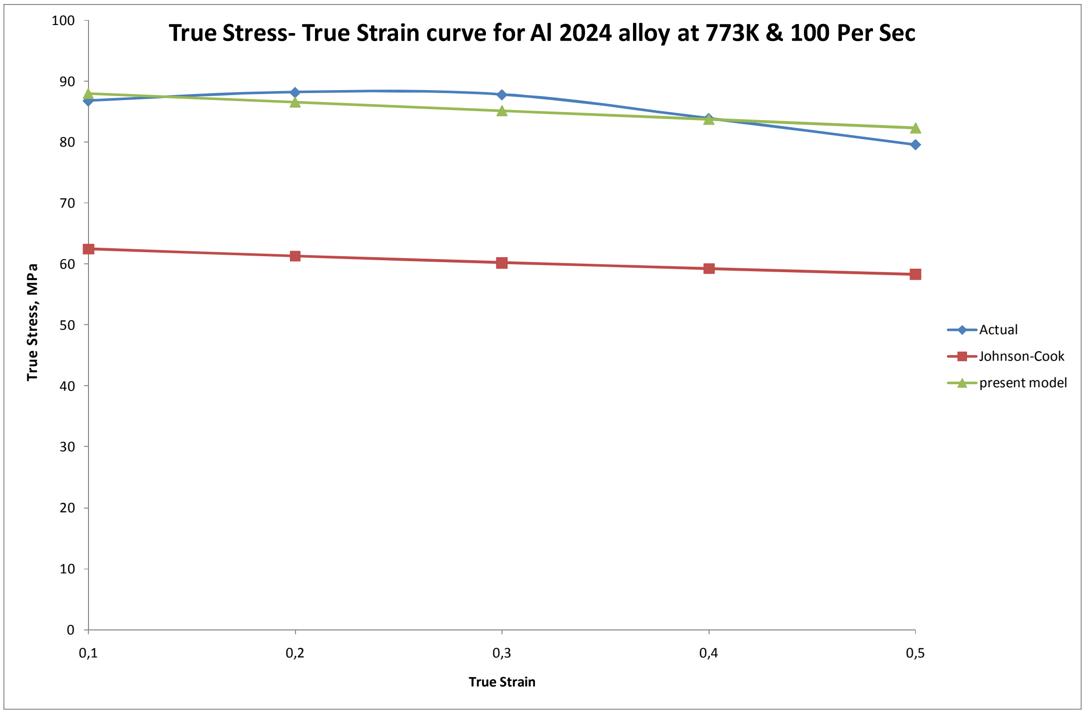
(a)


True Stress- True Strain curve for Al 2024 alloy at 623K & 1 Per Sec
True Stress, MPa
140
120
100
80
60
40
20
0
0,1
0,2
0,3
0,4
0,5
True Strain
Actual
Johnson-Cook
present model

(b)


True Stress- True Strain curve for Al 2024 alloy at 723K & 10 Per Sec
True Stress, MPa
100
90
80
70
60
50
40
30
20
10
0
0,1
0,2
0,3
0,4
0,5
True Strain
Actual
Johnson-Cook
present model

(c)



(d)

Fig-2: Comparison between the results at different temperature & Strain rate

Alloy	*κ*	***A***	***m***	**C**
Al-0.1Mg	5.033	-8.572	0.180	1.363
Al-0.5Mg	13.873	-7.894	0.090	1.287
Al-1.0Mg	24.697	-7.798	0.102	1.345
Al-2.0Mg	22.954	-6.525	0.097	1.311
Al-5.0Mg	42.193	3.997	0.099	1.346

TABLE- 2. VALUE OF CONSTANTS OBTAINED FOR PROPOSED MODEL

5. CONCLUSION

Analysis carried out in this research article shows the direct dependency of the flow stress on Strain, Strain rate & temperature. The main conclusions are as follows,

1. This model is completely dimensionless. So any parametric dimensions, whether standard or non-standard can be used.
2. This formula needs only 4 numbers of constants to define a flow stress value, thus easy to calculate the flow stress value.

3. The proposed model uses the results of hot compression test. Hence, it is more desirable as compared to JC model that needs the input from compression & torsion tests.
4. It reduces the number of constants from 48 in N.S. Babu Model to 4. Further, this model lessens two constants of Modified JC Model & Series Expansion Method, thus add another fact to its benefit.
5. Extrapolation/ interpolation can be made to estimate the values of flow stress at stringent experimental conditions.
6. It can be used to eliminate the experimental error due to sudden change in physical conditions like voltage fluctuation.
7. The proposed model covers a wider range of materials exhibiting the similar compressive behavior under the deforming compressive load.

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