A NEW APPROACH TO M(G)-GROUP SOFT UNION ACTION AND ITS APPLICATIONS TO M(G)-GROUP THEORY

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ABSTRACT:

In this paper, we define a new type of M(G)-group action , called M(G)-group soft union(SU) action and M(G)-ideal soft union(SU) action on a soft set. This new concept illustrates how a soft set effects on an M(G)-group in the mean of union and inclusion of sets and its function as bridge among soft set theory, set theory and M(G)-group theory. We also obtain some analog of classical M(G)- group theoretic concepts for M(G)-group SU-action. Finally, we give the application of SU-actions on M(G)-group to M(G)-group theory.

KEYWORDS:

soft set, M(G)-group, M(G)-group SU-action, M(G)--ideal SU-action, soft pre-image, soft anti-image, α-inclusion.

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1.INTRODUCTION:

Soft set theory as in [1, 2, 11, 14, 15, 16, 18, 25, 28] was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations Moreover, Atagun and Sezgin [4] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29] Sezgin et.al [25] introduced a new concept to the literature of N-group called N-group soft intersection action. Operations of soft sets have been studied by some authors, too. of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. In this paper, we define a new type of M(G)-group action on a soft set, which we call M(G)- group soft union action and abbreviate as "*M(G)-group SU action* "which is based on the inclusion relation and union of sets. Since M(G)- group

SU-action gathers soft set theory and set theory and M(G) –group theory, it is useful in improving the soft set theory with respect to M(G)- group structures. Based on this new notion, we then introduce the concepts of M(G)-ideal SU-action and show that if M(G)-group SU-action over U. Moreover, we investigate these notions with respect to soft image, soft pre-image and give their applications to M(G)- group theory.

2.PRELIMINARIES:

In this section, we recall some basic notions relevant to M(G)- groups and soft sets. By a near-ring, we shall mean an algebraic system (M(G),+, •),
where

(N_1) (M(G), +) forms a group (not necessarily abelian)
(N_2) (M(G), •) forms a semi group and
(N_3) (x + y)z = xz + yz for all x,y,z ∈ G.

Throughout this paper, M(G) will always denote right near-ring. A normal subgroup H of M(G) is called a left ideal of M(G) if g(f+i)-gf ∈ H for all g,f ∈ M(G) and i ∈ I and denoted by $H \lhd_\ell M(G)$. For a near-ring M(G), the zero-symmetric part of M(G) denoted by $M_0(G)$ is defined by $M_0(G)$ ={g∈ S / g0=0}.

Let G be a group, written additively but not necessarily abelian, and let M(G) be the set { f / f : G→ G } of all functions from G to G. An addition operation can be defined on M(G) ; given f,g in M(G), then the mapping f+g from G to G is given by (f+g)x = f(x) +g(x) for x in G.Then (M(G), +) is also group, which is abelian if and only if G is abelian. Taking the composition of mappings as the product, M(G) becomes a near-ring.

Let G be a group. Then, under the operation below;
μ : M(G) x G → G
(f,a) → fa
(G, μ) is called M(G)-group. Let M(G) be a near-ring, G_1 and G2 two M(G)-groups. Then $\phi : G_1 \rightarrow G_2$ is called M(G)- homomorphism if for all x,y ε G_1, for all g ε M(G),

(i) Φ (x+y) = ϕ(x) + ϕ(y)
(ii) Φ (gx) = g ϕ(x). It is denoted by G. Clearly M(G) itself is an M(G)-group by natural operations.

For all undefined concepts and notions we refer to (24). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A,B,C⊆ E.

2.1.Definition[22]: A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11,17,18,26,29,7].

2.2.Definition[6] :The relative complement of the soft set F_A over U is denoted by F^r_A, where $F^r_A : A \rightarrow P(U)$ is a mapping given as $F^r_A(a) = U \setminus F_A(a)$, for all a ∈ A.

2.3.Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by $F_A \Cap G_B$, and is defined as $F_A \Cap G_B = (H,C)$, where $C = A \cap B$ and for all c ∈ C, $H(c) = F(c) \cap G(c)$.

2.4.Definition[6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H,C)$,where $C = A \cap B$ and for all c ∈ C, $H(c) = F(c) \cup G(c)$.

2.5 Definition[12]: Let F_A and G_B be soft sets over the common universe U and ψbe a function from A to B. Then we can define the soft set ψ (F_A) over U, where $\psi (F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi (F_A)(b) = \cup\{F(a) \mid a \in A \text{ and } \psi (a) = b\}$,

if$\psi^{-1}(b) \neq \emptyset$, = 0 otherwise for all b ∈ B. Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi (a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.6.Definition[13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^\star(F_A)(b) = \cap\{F(a) \mid a \in A \text{ and } \psi (a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$,

=0 otherwise for all b ∈ B. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.7 Definition [8]: Let f_A be a soft set over U and α be a subset of U. Then, lower α-inclusion of a soft set f_A, denoted by f^α_A, is defined as $f^\alpha_A = \{x \in A : f_A(x) \subseteq \alpha\}$

3. M(G) –GROUP SU-ACTION

In this section, we first define M(G)-group soft union action, abbreviated as M(G)-group SU-action with illustrative examples. We then study their basic results with respect to soft set operation.

3.1Definition: Let S be an M(G)- group and f_S be a fuzzy soft set over U, then f_S is called fuzzy SU-action on M(G)- group over U if it satisfies the following conditions;

(FS_UN-1) $f_S(x+y) \subseteq f_S(x) \cup f_S(y)$
(FS_UN-2) $f_S(-x) \subseteq f_S(x)$
(FS_UN-3) $f_S(gx) \subseteq f_S(x)$
For all x,y ∈ S and g ∈M(G).

3.1Example: Consider the near-ring module M(G) ={e,f,g,h},be the near-ring under the operation defined by the following table:

+	e	f	g	h
e	e	f	g	h
f	f	e	f+g	f+h
g	g	g+h	e	g+h
h	h	h+f	h+g	e

.	e	f	g	h
0	e	f	g	h
f	f	e	f•g	f •h
g	g	g •h	e	g •h
h	h	h •f	h •g	e

Let G=M(G) be the set of functions and

and U= $\left\{\begin{bmatrix} f & e \\ f & e \end{bmatrix} / \text{f, e} \in \text{M(G)}\right\}$,2×2 matrices with four terms, is the universal set .we construct a soft set.

f_s (e)= $\left\{\begin{bmatrix} e & 0 \\ e & 0 \end{bmatrix},\right\}$, f_s (f)= f_s (g)= f_s (h)= $\left\{\begin{bmatrix} f & 0 \\ f & 0 \end{bmatrix},\begin{bmatrix} g & 0 \\ g & 0 \end{bmatrix},\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}\right\}$,

Then one can easily show that the soft set f_s is a M(G)-group SU-action over U.

3.1 Proposition: Let f_s be a fuzzy SU-action on M(G)- group over U. Then, f_s(0)⊆f_s(x) for all x ∈ S.

Proof: Assume that f_s is fuzzy SU-action over U.Then, for all x∈S ,
f_s(0)= f_s(x-x)⊆ f_s(x)∪ f_s(-x)= f_s(x)∪ f_s(x)= f_s(x).

3.1 Theorem: Let S be a fuzzy SU-action on M(G)- group and f_s be a fuzzy soft set over U. Then f_s is SU-action of M(G)- group over U if and only if

(i) f_s(x-y) ⊆ f_s(x) ∪ f_s(y)
(ii) f_s(gx) ⊆ f_s(x) for all x,y ∈ S and g ∈ M(G).

Proof: Suppose f_s is a fuzzy SU-action on M(G)- group over U. Then, by definition-3.1,
f_s(xy) ⊆ f_s(y) and f_s(x-y) ⊆ f_s(x) ∪ f_s(-y) = f_s(x) ∪ f_s(y) for all x,y ∈ S

Conversely, assume that f_s(xy) ⊆ f_s(y) and f_s(x-y) ⊆ f_s(x) ∪ f_s(y) for all x,y ∈ S.

If we choose x=0,then f_s(0-y)= f_s(-y) ⊆ f_s(0) ∪ f_s(y) =f_s(y) by proposition-3.1. Similarly

f_s(y)= f_s(−(-y)) ⊆ f_s(-y),thus f_s(-y) = f_s(y) for all y ∈ S. Also, by assumption
f_s(x-y) ⊆ f_s(x) ∪ f_s(-y) = f_s(x) ∪ f_s(y). This complete the proof.

3.2Theorem: Let f_s be a fuzzy SU-action on M(G)- group over U.
(i) If f_s(x-y)= f_s(0) for any x,y ∈ S ,then f_s(x) = f_s(y).
(ii) f_s(x-y)= f_s(0) for any x,y ∈ S ,then f_s(x) = f_s(y).

Proof: Assume that f_s(x-y)= f_s(0) for any x,y ∈ S ,then

f_s(x) = f_s(x-y+y) ⊆ f_s(x-y) ∪ f_s(y)

=f_s(0) ∪ f_s(y) =f_s(y)

and similarly,

f_s(y) = f_s((y-x)+x) ⊆ f_s(y-x) ∪ f_s(x)

= f_s(−(y-x)) ∪ f_s(x)

=f_s(0) ∪ f_s(x) =f_s(x)

Thus, f_s(x) =f_s(y) which completes the proof .Similarly, we can show the result (ii).
It is known that if S is an M(G)- group, then (S,+)is a group but not necessarily abelian.
That is, for any x,y ∈ S, x+y needs not be equal to y+x. However, we have the following:

3.3Theorem: Let f_s be fuzzy SU-action on M(G)- group over U and x ∈ S. Then,

f_s(x) =f_s(0)⇔ f_s(x+y) = f_s(y+x) = f_s(y) for all y ∈ S .

Proof: Suppose that f_s(x+y)= f_s(y+x) = f_s(y) for all y ∈ S. Then, by choosing y =0, we obtain that f_s(x) =f_s(0).

Conversely, assume that f_s(x) =f_s(0). Then by proposition-3.1,we have

f_s(0) =f_s(x) ⊂ f_s(y) , ∀ y ∈ S…………… (1)

Since f_s is fuzzy SU-action on N-module over U, then

f_s(x+y) ⊆ f_s(x) ∪ f_s(y) = f_s(y), ∀ y ∈ S. Moreover, for all y ∈ S

f_s(y) = f_s((-x)+x)+y) = f_s(-x+(x+y)) ⊆ f_s(-x) ∪ f_s(x+y)

= f_s(x) ∪ f_s(x+y)= f_s(x+y)

Since by equation (1), f_s(x) ⊆ f_s(y) for all y ∈ S and x,y ∈ S ,implies that x+y ∈ S .Thus, it follows that f_s(x) ⊆ f_s(x+y). So f_s(x+y) = f_s(y) for all y ∈ S.

Now, let x ∈ S. Then, for all x, y ∈ S

f_s(y+x) = f_s(y+x+(y-y))

= f_s(y+(x+y)-y)

⊆ f_s(y) ∪ f_s(x+y) ∪ f_s(y)

= f_s(y) ∪ f_s(x+y)= f_s(y)

Since f_s(x+y) = f_s(y). Furthermore, for all y ∈ S

f_s(y) = f_s(y+(x-x))

= f_s((y+x)-x)

⊆ f_s(y+x) ∪ f_s(x)

= f_s(y+x) by equation(1).

It follows that f_s(y+x) = f_s(y) and so f_s(x+y)= f_s(y+x)= f_s(y) , for all y ∈ S ,which completes the proof.

3.4Theorem: Let S be a near-field and $f_s$be a fuzzy soft set over U. If f_s(0)⊆ f_s(1)= f_s(x) for all 0≠ x ∈ S, then it is fuzzy SU-action on M(G)- group over U.

Proof: Suppose that $f_s(0) \subseteq f_s(1) = f_s(x)$ for all $0 \neq x \in S$. In order to prove that it is fuzzy SU-action on M(G)- group over U,it is enough to prove that $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$ and $f_s(gx) \subseteq f_s(x)$.

Let x, y ∈ S. Then we have the following cases:

Case-1: Suppose that x≠ 0 and y=0 or x=0 and y≠ 0. Since S is a near-field, so it follows that gx=0 and $f_s(gx) = f_s(0)$.since $f_s(0) \subseteq f_s(x)$,for all x ∈ S ,so$f_s(nx) = f_s(0) \subseteq f_s(x)$,and $f_s(nx) = f_s(0) \subseteq f_s(y)$. This imply $f_s(gx) \subseteq f_s(x)$.

Case-2: Suppose that x≠ 0 and y≠0. It follows that nx≠0.Then, $f_s(nx) = f_s(1) = f_s(x)$ and $f_s(gx) = f_s(1) = f_s(y)$,which implies that $f_s(gx) \subseteq f_s(x)$.

Case-3: suppose that x= 0 and y=0,then clearly $f_s(gx) \subseteq f_s(x)$.Hence $f_s(gx) \subseteq f_s(x)$, for all x,y ∈ S.

Now, let x,y ∈ S. Then x-y=0 or x-y≠ 0. If x-y=0,then either x=y=0 or x≠ 0,y≠ 0 and x=y. But, since $f_s(x-y) = f_s(0) \subseteq f_s(x)$, for all x ∈ S, it follows that $f_s(x-y) = f_s(0) \subseteq f_s(x) \cup f_s(y)$. If x-y≠0,then either x≠ 0,y≠ 0 and x≠y or x≠ 0 and y=0 or x=0 and y≠ 0.

Assume that x≠ 0,y≠ 0 and x≠y.This follows that

$$f_s(x-y) = f_s(1) = f_s(x) \subseteq f_s(x) \cup f_s(y).$$

Now, let x≠ 0 and y=0. Then $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$. Finally, let x=0 and y≠ 0. Then, $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$. Hence $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$, for all x, y ∈ S. Thus, f_s is fuzzy SU-action on M(G)- group over U.

3.5Theorem: Let f_s and f_T be two fuzzy SU-action on M(G)- group over U. Then $f_s \wedge f_T$ is fuzzy soft SU-action on M(G)- group over U.

Proof: let (x_1, y_1), (x_2, y_2) ∈ S×T.Then

$$\begin{aligned} f_{S\wedge T}\left(\left(x_{1,}y_1\right) - \left(x_{2,}y_2\right)\right) &= f_{S\wedge T}\left(x_{1-}x_{2,}\ y_{1-}y_2\right) \\ &= f_S\left(x_{1-}x_2\right) \cap\ f_T\left(y_{1-}y_2\right) \\ &\subseteq \left(f_S\left(x_1\right) \cup f_S\left(x_2\right)\right) \cap \left(f_T\left(y_1\right) \cup f_T\left(y_2\right)\right) \\ &= \left(f_S\left(x_1\right) \cup f_T\left(y_1\right)\right) \cap \left(f_S\left(x_2\right) \cup f_T\left(y_2\right)\right) \\ &= f_{S\wedge T}\left(x_{1,}y_1\right)\ \cap\ f_{S\wedge T}\left(x_{2,}y_2\right) \end{aligned}$$

and

$$\begin{aligned} f_{S\wedge T}\left(\left(g_{1,}g_2\right), \left(x_{2,}y_2\right)\right) &= f_{S\wedge T}\left(n_1x_{2,}n_2y_2\right) \\ &= f_S\left(g_1x_2\right) \cap f_T\left(g_2y_2\right) \\ &\subseteq f_S\left(x_2\right) \cap f_T\left(y_2\right) \\ &= f_{S\wedge T}\left(x_{2,}y_2\right) \end{aligned}$$

Thus $f_s \wedge f_T$ is fuzzy SU-action on M(G)- group over U.
Note that $f_s \vee f_T$ is not fuzzy SU-action on M(G)- group over U.

3.2Example: Assume U = p_3 is the universal set. Let S = Z_3 and H =$\left\{\begin{bmatrix} a & a \\ b & b \end{bmatrix} / a, b \in Z_3\right\}$ 2× 2 matrices with Z_3 terms, be set of parameters. We define fuzzy SU-action on M(G)- group f_S over U= p_3 by

$f_S(0) = p_3$
$f_S(1) = \{(1), (1\ 2), (1\ 3\ 2)\}$
$f_S(2) = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$

We define fuzzy SU-action on N-module $f_H$over U= p_3 by
$f_H\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\} = p_3$
$f_H\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right\} = \{(1), (1\ 2), (1\ 3\ 2)\}$

Then $f_s \vee f_T$ is not fuzzy SU-action on M(G)- group over U.

3.2Definition: Let $f_{S,}\ g_T$ be fuzzy SU-action on M(G)- group over U. Then product of fuzzy

SU-action on M(G)- group f_S and g_T is defined as $f_S \times g_T = h_{S\times T}$, where
$h_{S\times T}(x, y) = f_S\,(x) \times\ g_T$(y) for all (*x*,*y*) ∈ S × T.

3.6Theorem: If f_S *and* g_T are fuzzy SU-action on M(G)- group over U. Then so is $f_S \times g_T$ over U×U.

Proof: By definition-3.2, let $f_S \times g_T$ =$h_{S\times T}$, where $h_{S\times T}(x, y) = f_S\,(x) \times\ g_T$(y) for all (*x*,*y*) ∈ S × T. Then for all (x_1, y_1), (x_2, y_2) ∈ S × T and $(n_{1,} n_2)$=N×N.

$$\begin{aligned} h_{S\times T}\left((x_{1,}y_1) - (x_{2,}y_2)\right) &= h_{S\times T}\,(x_{1-}x_{2,}\ y_{1-}y_2) \\ &= f_S\,(x_{1-}x_2\,) \times\ \ g_T\,(y_{1-}y_2\,) \\ &\subseteq (f_S\,(x_1\,) \cup f_S\,(x_2\,)) \times (g_T\,(y_1\,) \cup g_T\,(y_2\,)) \\ &=(f_S\,(x_1\,) \times g_T\,(y_1\,)) \cup (f_S\,(x_2\,) \times g_T\,(y_2\,)) \\ &= h_{S\times T}(x_{1,}y_1)\ \cup\ h_{S\times T}(x_{2,}y_2) \end{aligned}$$

$$\begin{aligned} h_{S\times T}\left((g_{1,}g_2)(x_{2,}y_2)\right) &= h_{S\times T}(n_1x_{2,}n_2y_2) \\ &= f_S(g_1x_2) \times g_T\,(g_2y_2\,) \\ &\subseteq f_S(x_2) \times g_T\,(y_2\,) \\ &= h_{S\times T}(x_{2,}y_2) \end{aligned}$$

Hence $f_S \times g_T$ =$h_{S\times T}$ is fuzzy SU-action on M(G)- group over U.

3.7Theorem: If f_S and h_S are fuzzy SU-action on M(G)-group over U,then so is $f_S \tilde{\cap} h_S$ over U.

Let *x*, *y* ∈ s and n ∈ N then

$$\begin{aligned} (f_S \tilde{\cap} h_S)\ (x\text{-}y) &= f_S\ (x\text{-}y) \cap h_S\ (x\text{-}y) \\ &\subseteq (f_S\,(x\,) \cup f_S\,(y\,)) \cap (h_S\,(x\,) \cup h_S\,(y\,)) \\ &= (f_S\,(x\,) \cap\ h_S(x\,)) \cup (f_S\,(y\,) \cap\ h_S\,(y\,)) \\ &=\ (f_S \tilde{\cap} h_S)\,(x) \cup\ \ (f_S \tilde{\cap} h_S)(y) \end{aligned}$$

$(f_S \tilde{\cap} h_S)(nx) = f_S(nx) \cap h_S(nx) \subseteq f_S(x) \cap h_S(x) = (f_S \tilde{\cap} h_S)(x)$

Therefore, $(f_S \tilde{\cap} h_S)$ is fuzzy SU-action on M(G)- group over U.

4.SU-ACTION ON M(G)-IDEAL STRUCTURES

4.1 Definition : Let S be an M(G)- group and f_S be a fuzzy soft set over U. Then f_S is called fuzzy SU-action on M(G)-ideal of S over U if the following conditions are satisfied:

(i) $f_s(x+y) \subseteq f_s(x) \cup f_s(y)$
(ii) $f_s(-x) = f_s(x)$
(iii) $f_s(x+y-x) \subseteq f_s(y)$
(iv) $f_s(g(x+y)-gx) \subseteq f_s(y)$ for all $x, y \in$ S and $g \in$ M(G).

Here, note that
$f_s(x+y) \subseteq f_s(x) \cup f_s(y)$ and $f_s(-x) = f_s(x)$ imply $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$

4.1Example: Consider M(G) $=\{0, \mathrm{x}, \mathrm{y}, \mathrm{z}\}$ with the following tables

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

.	0	x	y	z
0	0	0	0	0
x	0	0	0	x
y	0	x	y	y
z	0	x	y	z

Let S=M(G) be the parameters and U= D_2 ,dihedral group, be the universal set. We define a fuzzy soft set f_s over U by $f_s(0) = D_2$, $f_s(x) = \{e, b, ba\}$, $f_s(y) = \{a, b\}$, $f_s(z) = \{b\}$.
Then, one can show that f_s is fuzzy SU-action on M(G)-ideal of S over U.

4.2 Example: Consider the near –ring N={0,1,2,3} with the following tables

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	x	y	z
0	0	0	0	0
x	0	1	0	1
y	0	3	0	3
z	0	2	0	2

Let S=M(G) be the set of parameters and U= Z^+ be the universal set. We define a fuzzy soft set

f_s over U by $f_s(0) = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 17\}$
$f_s(1) = f_s(3) = \{1, 3, 5, 7, 9, 11\}$,$f_s(2) = \{1, 5, 7, 9, 11\}$

Since $f_s(2.(3+1)-2.3) = f_s(2.1-2.3) = f_s(3-3) = f_s(0) \nsubseteq f_s(1)$
Therefore, f_s is not fuzzy SU-action on M(G)-ideal over U. It is known that if M(G) is a zero-symmetric near-ring, then every M(G)-ideal of S is also M(G)- group of S. Here, we have an analog for this case.

4.1Theorem: Let M(G) be a zero- symmetric near-ring. Then, every fuzzy SU-action on M(G)-ideal is fuzzy SU-action on M(G)- group over U.

Proof: Let f_s be an fuzzy SU-action on M(G)-ideal on S over U. Since f_s(g(x+y)-gx) ⊆ f_s(y), for all x,y ∈ S, and g ∈ M(G), in particular for x=0, it follows that f_s (g(0+y)-g.0) = f_s (ny-0)= f_s(y)⊆ f_s(y). Since the other condition is satisfied by definition-4.1, f_s is fuzzy SU-action on M(G)-ideals of S over U.

4.2Theorem: Let f_s be fuzzy SU-action on M(G)-ideal of S and f_T be fuzzy SU-action on M(G)-ideal of T over U. Then $f_s \wedge f_T$ is fuzzy SU-action on M(G)-ideal of S×T over U.

4.3 Theorem : If f_s is fuzzy SU-action on M(G)-ideal of S and f_T be fuzzy SU-action on M(G)-ideal of T over U, then $f_s \times f_T$ is fuzzy SU-action on M(G)-ideal over U×U.

4.4 Theorem : If f_s and h_s are two fuzzy SU-action on M(G)- group of S over U, then $f_S \,\widetilde{\cap}\, h_S$ is Fuzzy SU-action on M(G)-ideal over U.

5.APPLICATION OF FUZZY SU-ACTION ON M(G)- GROUP

In this section, we give the applications of fuzzy soft image, soft pre-image, lower α-inclusion of fuzzy soft sets and M(G)- group homomorphism with respect to fuzzy SU-action on M(G)- group and M(G)-ideal.

5.1Theorem: If f_s is fuzzy SU-action on M(G)-ideal of S over U, then S^f={x ∈S / f_s(x)= f_s(0)} is a M(G)-ideal of S.

Proof: It is obvious that 0 ∈ S^f we need to show that (i) x-y ∈ S^f, (ii) s+x-s ∈ S^f and
(iii) g(s+x)-gs ∈ S^f for all x,y ∈ S^f and g∈M(G) and s∈S.

If x,y∈ S^f, then f_s(x)= f_s(y) = f_s(0). By proposition-3.1,
f_s(0) ⊆ f_s(x-y), f_s(0) ⊆ f_s(s+x-s), and f_s(0) ⊆ f_s(n(s+x)-ns) for all x,y ∈ S^f and g∈M(G) and s∈S. Since f_s is fuzzy SU-action on M(G)-ideal of S over U, then for all x,y ∈ S^f and g∈M(G) and s∈S.

(i) f_s(x-y) ⊆ f_s(x)∪ f_s(y) = f_s(0).
(ii) f_s(s+x-s)⊆ f_s(x) =f_s(0).
(iii)f_s(g(s+x)-gs) ⊆ f_s(x) =f_s(0).

Hence f_s(x-y) = f_s(0), f_s(s+x-s)=f_s(0) and f_s(g(s+x)-gs) =f_s(0), for all x,y ∈ S^f and g∈M(G) and s∈S. Therefore S^f is M(G)-ideal of S.

5.2Theorem: Let f_s be fuzzy soft set over U and α be a subset of U such that $\emptyset \supseteq \alpha \supseteq f_s(0)$. If f_s is fuzzy SU-action on M(G)-ideal over U, then ${f_s}^{\subseteq\alpha}$is an M(G)-ideal of S.

Proof: Since $f_s(0) \subseteq \alpha$, then $0 \in {f_s}^{\subseteq\alpha}$ and $\emptyset \neq {f_s}^{\subseteq\alpha} \supseteq$ S. Let x,y $\in {f_s}^{\subseteq\alpha}$, then f_s(x) $\subseteq \alpha$ and f_s(y) $\subseteq \alpha$. We need to show that

(i) x-y $\in {f_s}^{\subseteq\alpha}$
(ii) s+x-s $\in {f_s}^{\subseteq\alpha}$
(iii) g(s+x)-ns $\in {f_s}^{\subseteq\alpha}$ for all x,y $\in {f_s}^{\subseteq\alpha}$and g∈M(G) and s∈S.

Since f_s is fuzzy SU-action on M(G)-ideal over U, it follows that

(i) f_s(x-y) $\subseteq f_s$(x)∪ f_s(y) $\subseteq \alpha \cup \alpha = \alpha$,
(ii) f_s(s+x-s)$\subseteq f_s$(x) $\subseteq \alpha$ and
(iii)f_s(g(s+x)-gs) $\subseteq f_s$(x) $\subseteq \alpha$. Thus, the proof is completed.

5.3 Theorem : Let f_s and f_T be fuzzy soft sets over U and χ be an M(G) -isomorphism from S to T.If f_s is fuzzy SU-action on M(G)-ideal of S over U, then $\chi(f_s)$ is fuzzy SU-action on M(G)-ideal of T over U.

Proof: Let δ_1, δ_2 and g∈ M(G). Since χ is surjective, there exists $s_1, s_2 \in$ S such that $\chi(s_1) = \delta_1$ and $\chi(s_2) = \delta_2$. Then

$$(\chi f_s)(\delta_1 - \delta_2) = \cup \{ f_s(s) / s \in S, \chi(s) = \delta_1 - \delta_2 \}$$
$$= \cup \{ f_s(s) / s \in S, s = \chi^{-1}(\delta_1 - \delta_2) \}$$
$$= \cup \{ f_s(s) / s \in S, s = \chi^{-1}(\chi(s_1 - s_2)) = s_1 - s_2 \}$$
$$= \cup \{ f_s(s_1 - s_2) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\ldots \}$$
$$\subseteq \cup \{ f_s(s_1) \cup f_s(s_2)) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\ldots \}$$
$$= \left(\{\cup \{f_s(s_1)/s_1 \in S, \chi(s_1) = \delta_1\}\} \cup \{\cup \{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2\}\}\right)$$
$$= (\chi(f_s))(\delta_1) \cup (\chi(f_s))(\delta_2)$$

Also $(\chi f_s)(\delta_1 + \delta_2 - \delta_1) = \cup \{ f_s(s) / s \in S, \chi(s) = \delta_1 + \delta_2 - \delta_1 \}$

$$= \cup \{ f_s(s) / s \in S, s = \chi^{-1}(\delta_1 + \delta_2 - \delta_1) \}$$
$$= \cup \{ f_s(s) / s \in S, s = \chi^{-1}(\chi(s_1 + s_2 - s_1)) = s_1 + s_2 - s_1 \}$$
$$= \cup \{ f_s(s_1 + s_2 - s_1) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\ldots \}$$
$$\subseteq \cup \{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2 \}$$
$$= (\chi(f_s))(\delta_2)$$

Furthermore, $(\chi f_s)(g(\delta_1 + \delta_2) - g\delta_1) = \cup \{ f_s(s) / s \in S, \chi(s) = g(\delta_1 + \delta_2) - g\delta_1 \}$

$$= \cup \{ f_s(s) / s \in S, s = \chi^{-1}(g(\delta_1 + \delta_2) - g\delta_1) \}$$
$$= \cup \{ f_s(s) / s \in S, s = g(s_1 + s_2) - gs_1 \}$$
$$= \cup \{ f_s(g + s_2) - gs_1) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\ldots \}$$
$$\subseteq \cup \{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2 \}$$
$$= (\chi(f_s))(\delta_2).$$

Hence $\chi(f_s)$ is fuzzy SU-action on M(G)-ideal of T over U.

5.4Theorem: Let f_S and f_T be fuzzy soft sets over U and χ be an M(G)--isomorphism from S to T. If f_T is fuzzy SU-action on M(G)-ideal of T over U, then $\chi^{-1}(f_T)$ is fuzzy SU-action on M(G)-ideal of S over U.

Proof: Let s_1, s_2 ∈S and g∈ M(G). Then

$$(\chi^{-1}(f_T))\,(s_1 - s_2) = f_T(\chi\,(s_1 - s_2))$$
$$= f_T(\chi\,(s_1)\text{-}\chi(s_2))$$
$$\subseteq f_T(\chi\,(s_1)) \cup f_T(\chi\,(s_2))$$
$$= (\chi^{-1}(f_T))(s_1) \cup (\chi^{-1}(f_T))(s_2).$$

Also $(\chi^{-1}(f_T))\,(s_1 + s_2 - s_1) = f_T(\chi\,(s_1 + s_2 - s_1))$

$$= f_T(\chi\,(s_1) + \chi(s_2) - \chi(s_1))$$
$$\subseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))\,(s_2)$$

Furthermore, $(\chi^{-1}(f_T))\,(\mathrm{g}(s_1 + s_2) - \mathrm{g}s_1) = f_T(\chi\,(\mathrm{g}(s_1 + s_2) - \mathrm{g}s_1))$

$$= f_T(\mathrm{g}(\chi\,(s_1) + \chi(s_2)) - \mathrm{g}\chi(s_1))$$
$$\subseteq f_T(\chi\,(s_2)) = (\chi^{-1}(f_T))\,(s_2)$$

Hence, $(\chi^{-1}(f_T))$ is fuzzy SU-action on M(G)-ideal of S over U.

CONCLUSION:

In this paper, we have defined a new type of N-module action on a fuzzy soft set, called fuzzy SU-action on M(G)- group by using the soft sets. This new concept picks up the soft set theory, fuzzy theory and M(G)- group theory together and therefore, it is very functional for obtaining results in the mean of M(G)- group structure. Based on this definition, we have introduced the concept of fuzzy SU-action on M(G)-ideal. We have investigated these notions with respect to soft image, soft pre-image and lower α-inclusion of soft sets. Finally, we give some application of fuzzy SU-action on M(G)-ideal to M(G)- group theory. To extend this study, one can further study the other algebraic structures such as different algebra in view of their SU-actions.

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