Bipolar Fuzzy Hypergraphs

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Abstract

In this paper, we define some basic concepts of bipolar fuzzy hypergraphs, cut level bipolar fuzzy hypergraphs, dual bipolar fuzzy hypergraphs and bipolar fuzzy transversal. Also some basic theorems related to the stated graphs have been presented.

Keywords:

Bipolar fuzzy hypergraphs, cut level bipolar fuzzy hypergraphs, bipolar fuzzy transversal.

1 Introduction

1.1 Fuzzy sets

In 1965, Zadeh [10] introduced the notion of fuzzy subset of a set as a method of presenting uncertainty. The fuzzy systems have been used with success in last years, in problems that involve the approximate reasoning. It has vast research area in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, signal processing, multiagent systems, pattern recognition, robotics, computer networks, expert systems, decision making and automata theory. A *fuzzy set* A on a set X is characterized by a mapping $\mu : X \rightarrow [0,1]$, called the membership function. We shall denote a fuzzy set as $A = (X, \mu)$.

1.2 Bipolar fuzzy sets

In 1994, Zhang [11] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose range of membership degree is $[-1,1]$. In

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bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0,1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1,0)$ of an element indicates the element somewhat satisfies the implicit counter property.

Let X be nonempty set. A *bipolar fuzzy set* B on X is an object having the form $B = \{(x, \mu^+(x), \mu^-(x)) \mid x \in X\}$, where $\mu^+ : X \to [0,1]$ and $\mu^- : X \to [-1,0]$ are mappings.

If $\mu^+(x) \neq 0$ and $\mu^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. If $\mu^+(x) = 0$ and $\mu^-(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^+(x) \neq 0$ and $\mu^-(x) \neq 0$ when membership function of the property overlaps that of its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $B = (\mu^+, \mu^-)$ for the bipolar fuzzy set $B = \{(x, \mu^+(x), \mu^-(x)) \mid x \in X\}$. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. For instance, when we assess the position of an object in space, we may have positive information expressed as a set of possible places and negative information expressed as a set of impossible places. This corresponds to the idea that the union of positive and negative information does not cover the whole space.

Height [8] of a bipolar fuzzy set $B = \{(x, \mu^+(x), \mu^-(x)) \mid x \in X\}$ of a nonempty set X is denoted by $h(B)$ and defined as $h(B) = max\{\mu^+(x) \mid x \in X\}$. *Depth* [8] of a bipolar fuzzy set B of a nonempty set X is denoted by $d(B)$ and defined as $d(B) = min\{\mu^-(x) \mid x \in X\}$. Let $B_1 = \{(x, \mu_1^+(x), \mu_1^-(x)) \mid x \in X\}$ and $B_2 = \{(x, \mu_2^+(x), \mu_2^-(x)) \mid x \in X\}$ be two bipolar fuzzy sets in X. $B_1 \subseteq B_2$ if $\mu_1^+(x) \leq \mu_2^+(x)$ for all $x \in X$ and $\mu_1^-(x) \geq \mu_2^-(x)$ for all $x \in X$. The *support* [8] of B is denoted by $supp(B)$ and defined by $supp(B) = \{x \mid \mu^+(x) \neq 0$ or $\mu^-(x) \neq 0\}$. The *upper core* [8] of B is denoted by $\bar{c}(B)$ and defined by $\bar{c}(B) = \{x \mid \mu^+(x) = 1\}$. Similarly, the *lower core* [8] of B is denoted by $\underline{c}(B)$ and defined by $\underline{c}(B) = \{x \mid \mu^-(x) = -1\}$. Let $t_1 \in (0,1]$, $t_2 \in [-1,0)$ and $B = (\mu^+, \mu^-)$ be a bipolar fuzzy set. $\{t_1, t_2\}$ *cut level set* [8] of B to be the crisp set $B_{t_2}^{t_1} = \{x \in supp(B) \mid \mu^+(x) \geq t_1$ and $\mu^-(x) \leq t_2\}$.

For every two bipolar fuzzy sets $A = (\mu_A^+, \mu_A^-)$ and $B = (\mu_B^+, \mu_B^-)$ on X,

$(A \cap B)(x) = (min(\mu_A^+(x), \mu_B^+(x)), max(\mu_A^-(x), \mu_B^-(x)))$.

$(A \cup B)(x) = (max(\mu_A^+(x), \mu_B^+(x)), min(\mu_A^-(x), \mu_B^-(x)))$.

1.3 Hypergraphs

A (crisp) *hypergraph* on a set X is a pair $H^* = (X, E)$ where X is a finite set and E is a finite family of nonempty subsets of X which satisfy the condition: Every member of X is contained in some member of E. X is called the vertex set and E is the edge set of H^*. Multiple or repeated edges are allowed. A hypergraph $H^* = (X, E)$ is *simple* if E contains no repeated edges and whenever $E_1, E_2 \in E$ and $E_1 \subset E_2$, then $E_1 = E_2$. A hypergraph $H^* = (X; E_1, E_2, \ldots, E_k)$ where $X = \{x_1, x_2, \ldots, x_n\}$ can be mapped to a hypergraph $H^{**} = (E; x_1, x_2, \ldots, x_n)$ whose vertices are the points $e_1, e_2, \ldots, e_k$ (corresponding to $E_1, E_2, \ldots, E_k$), and whose edges are the sets $X_1, X_2, \ldots, X_n$ (corresponding to $x_1, x_2, \ldots, x_n$ respectively) where $X_j = \{x_j \in E_i, i \le k\}, j = 1,2,\ldots,n$. The hypergraph H^{**} is called the *dual hypergraph* of H.

Suppose $H_1^* = (X_1, E_1)$ and $H_2^* = (X_2, E_2)$ are crisp hypergraphs. Then H_1^* is *partial hypergraph* of H_2^* if $E_1 \subseteq E_2$, this relationship is denoted by $H_1^* \le H_2^*$. A sequence of crisp hypergraphs $H_i^* = (X_i, E_i)$, $1 \le i \le n$ is said to be *ordered* if $H_1 < H_2 < \ldots < H_n$. The sequence $\{H_i^* | 1 \le i \le n\}$ is *simply ordered* if it is ordered and if whenever $E \in E_{i+1} \setminus E_i$, then $E \not\subseteq X_i$.

1.4 Fuzzy hypergraphs

Let X be a finite set and let E be a finite family of nontrivial fuzzy sets on X (or subsets of X) such that $X = \bigcup\{suppA \,|\, A \in \mathrm{E}\}$. Then the pair $\mathrm{H} = (X, \mathrm{E})$ is a *fuzzy hypergraph* on X. X and E are respectively vertex set and fuzzy edge set of H. The height of H, $h(\mathrm{H})$, is defined by $h(\mathrm{H}) = max\{h(A) \,|\, A \in \mathrm{E}\}$. A fuzzy hypergraph is *simple* if E has no repeated fuzzy edges and whenever $A, B \in \mathrm{E}$ and $A \subseteq B$, then $A = B$. A fuzzy hypergraph $\mathrm{H} = (X, \mathrm{E})$ is *support simple* if whenever $A, B \in \mathrm{E}$, $A \subseteq B$ and $supp(A) = supp(B)$, then $A = B$. Suppose $A = (X_1, \mu) \in \xi, X_1 \subseteq X$ and $c \in (0,1]$. The $c-$cut of A, A^c, is defined by $A^c = \{x \in X \,|\, \mu(x) \ge c\}$. If $\mathrm{E}^c = \{A^c | \in \mathrm{E} / \{\phi\}$ and $X^c = \bigcup\{A^c \,|\, A \in \mathrm{E}\}$. If $\mathrm{E}^c \ne \phi$, then the (crisp) hypergraph $H^c = (X^c, \mathrm{E}^c)$ is the $c-$ *level hypergraph* of H.

Suppose $\mathrm{H}_1 = (X, \mathrm{E}_1)$ and $\mathrm{H}_2 = (X, \mathrm{E}_2)$ are fuzzy hypergraphs. Then H_1 is partial hypergraph of H_2 if $\mathrm{E}_1 \subseteq \mathrm{E}_2$. A fuzzy set $A = (X, \mu)$ with $\mu : X \to [0,1]$ is an *elementary fuzzy set* if μ is constant function or μ has range $\{0, a\}, 0 \ne a$. An *elementary fuzzy hypergraph* is a fuzzy hypergraph in which all fuzzy edges are elementary.

A fuzzy hypergraph $\mathrm{H} = (X, \mathrm{E})$ is a m *tempered fuzzy hypergraph* of a crisp hypergraph $H^* = (X, E)$ if there exists a fuzzy set $A = (X, m)$ such that $m : X \to (0,1]$ and $\mathrm{E} = \{\gamma_{E_i} \mid E_i \in E\}$ where

$$\gamma_{E_i}(x) = \begin{cases} min\{m(e) \mid e \in E_i\} & if \ \ x \in E_i \\ 0, & if \ \ otherwise \end{cases}$$

A *fuzzy transversal* $\mathrm{T} = (X, \tau)$ of H is a fuzzy set defined on X with the property that $\tau_{h(A)} \cap \mu_{h(A)} \neq \phi$ for each $A \in \mathrm{E}$ (recall that $h(A)$ is the height of A). A *minimal fuzzy transversal* T for H is a transversal of H with the property that if $T_1 < T$, then T_1 is not a fuzzy transversal of H.

1.5 Review of previous works

Samanta and Pal [6] introduced fuzzy threshold graphs. They also introduced fuzzy tolerance graphs [7], competition graphs [9] and bipolar fuzzy intersection and line graphs [8]. Akram [1] defined different operations on bipolar fuzzy graphs. Goetschel [2] introduced the concept of fuzzy hypergraphs and Hebbian structures. Goetschel [3] also explained the colorings of fuzzy hypergraphs. Goetschel [4] defined intersecting fuzzy hypergraphs. Samanta and Pal [8] introduced bipolar fuzzy intersection graphs and bipolar fuzzy line graphs.

1.6 Our works

In this paper we defined bipolar fuzzy hypergraphs, cut level sets and dual of bipolar fuzzy hypergraphs and fuzzy transversal.

2 Bipolar fuzzy hypergraphs

Definition 1 *Let X be a finite set and let ξ be a finite family of bipolar fuzzy subsets B on X (or subsets of X) such that $X = \bigcup_{B \in \xi} supp(B)$. The pair $H = (X, \xi)$ is called a bipolar fuzzy hypergraph (on X) and ξ is called edge set of H, which are bipolar fuzzy sets on subsets of X.*

The height of H, $h(H)$, is defined by $h(H) = max\{h(B) \mid B \in \xi\}$. The depth of H, $d(H)$, is defined by $d(H) = min\{d(B) \mid B \in \xi\}$. Here $\mu^+(x_i), \mu^-(x_i)$ are the positive and negative membership values for the edges of H. The incidence matrix of bipolar fuzzy hypergraph are of the form $(a_{ij}, \mu^+(x_i), \mu^-(x_i))$.

Example 1 We consider a bipolar fuzzy hypergraph $H = (X, \xi)$ such that

$X=\{x_1,x_2,x_3,x_4,x_5,x_6\}$, $\xi=\{B_1,B_2,B_3,B_4,B_5\}$. Here
$B_1=\{(x_1,0.4,-0.3),(x_2,0.5,-0.6),(x_3,0.6,-0.4)\}$,
$B_2=\{(x_2,0.5,-0.3),(x_3,0.4,-0.8),(x_4,0.6,-0.2)\}$, $B_3=\{(x_3,0.8,-0.4),(x_5,0.9,-0.4)\}$,
$B_4=\{(x_4,0.5,-0.4),(x_5,0.6,-0.2),(x_6,0.9,-0.5)\}$,
$B_5=\{(x_5,0.3,-0.5),(x_6,0.5,-0.7)\}$. The corresponding diagram is shown in Figure 1.

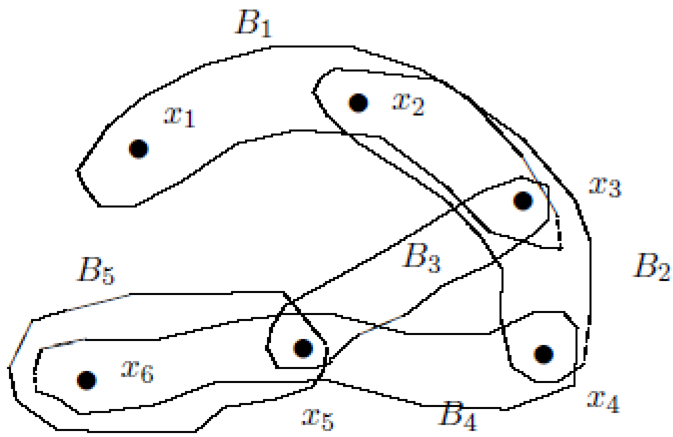


Figure 1: Example of a bipolar fuzzy hypergraph.

The corresponding incidence matrix M_H is given

$$M_H = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{pmatrix} (0.4,-0.3) & (0,0) & (0,0) & (0,0) & (0,0) \\ (0.5,-0.6) & (0.5,-0.3) & (0,0) & (0,0) & (0,0) \\ (0.6,-0.4) & (0.4,-0.8) & (0.8,-0.4) & (0,0) & (0,0) \\ (0,0) & (0.6,-0.2) & (0,0) & (0.5,-0.4) & (0,0) \\ (0,0) & (0,0) & (0.9,-0.4) & (0.6,-0.2) & (0.3,-0.5) \\ (0,0) & (0,0) & (0,0) & (0.9,-0.5) & (0.5,-0.7) \end{pmatrix} \end{array}$$

Definition 2 *A bipolar fuzzy hypergraph* $H=(X,\xi)$ *is simple if* ξ *has no repeated bipolar fuzzy edges and whenever* $A,B\in\xi$ *and* $A\subseteq B$, *then* $A=B$.

Definition 3 *A bipolar fuzzy hypergraph* $H=(X,\xi)$ *is support simple if whenever* $A,B\in\xi$ *and* $A\subseteq B$ *and* $supp(A)=supp(B)$, *then* $A=B$.

Definition 4 *Let* $H_1 = (X_1, \xi_1)$ *and* $H_2 = (X_2, \xi_2)$ *be two bipolar fuzzy hypergraphs.* H_1 *is called partial bipolar fuzzy hypergraph of* H_2 *if* $\xi_1 \subseteq \xi_2$.

Example 2 Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ be a finite set and $\xi = \{B_1, B_2, B_3, B_4\}$ be the bipolar fuzzy set on subsets of X. Here $B_1 = \{(x_1, 0.4, -0.3), (x_2, 0.6, -0.2), (x_3, 0.7, -0.4)\}$, $B_2 = \{(x_3, 0.6, -0.5), (x_4, 0.4, -0.7)\}$, $B_3 = \{(x_3, 0.9, -0.6), (x_5, 0.4, -0.2)\}$, $B_4 = \{(x_4, 0.8, -0.7), (x_5, 0.4, -0.1)\}$. The hypergraph (X, ξ) is a simple and support simple bipolar fuzzy hypergraph shown in Figure 2.

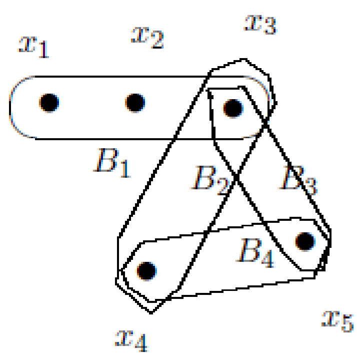
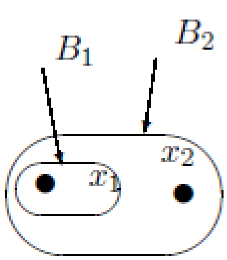


Figure 2: Example of simple and support simple bipolar fuzzy hypergraph.

Example 3 We now give some example of bipolar fuzzy edges which are absent in simple bipolar fuzzy hypergraph (in Figure 3) and support simple bipolar fuzzy hypergraph (in Figure 4)



$$B_1 = \{((x_1, 0.4, -0.5)\}$$
$$B_2 = \{(x_1, 0.8, -0.2), (x_2, 0.1, -0.3))\}$$

Figure 3: Bipolar fuzzy edges absent in simple bipolar fuzzy hypergraph.

$$B_1 = \{(x_1, 0.4, -0.5), (x_2, 0.7, -0.4), (x_3, 0.6, -0.3)\}$$
$$B_2 = \{(x_1, 0.5, -0.4), (x_2, 0.4, -0.6), (x_3, 0.5, -0.4)\}$$

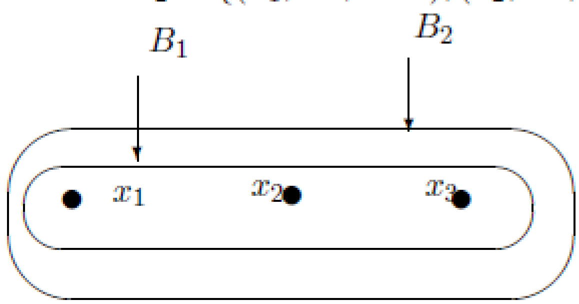


Figure 4: Bipolar fuzzy edges absent in support simple bipolar fuzzy hypergraph.

We are giving an example of a bipolar fuzzy hypergraph which is support simple but not simple. Let $H = (X, \xi)$ be a bipolar fuzzy hypergraph whose incidence matrix is given below.

$$M_H = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{pmatrix} (0.4, -0.3) & (1, -1) & (0, 0) & (0, 0) & (0.9, -0.8) \\ (0.5, -0.6) & (1, -1) & (1, -1) & (0.8, -0.6) & (0, 0) \\ (0, 0) & (0, 0) & (1, -1) & (0.6, -0.5) & (0.8, -0.5) \\ (0, 0) & (0.6, -0.2) & (0, 0) & (0.5, -0.4) & (0.8, -0.5) \end{pmatrix} \end{array}$$

Here $B_1 \subset B_2$ and $B_1 \neq B_2$. So H is not simple bipolar fuzzy hypergraph. But clearly it is support simple bipolar fuzzy hypergraph.

Definition 5 *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a non empty finite set and $B = \{B_1, B_2, \ldots, B_k\}$ be bipolar sets of subsets of X. (α, β) - cut of bipolar fuzzy hypergraph, $H = (X, B)$, denoted by $H_{(\alpha,\beta)}$, is an ordered pair $H_{(\alpha,\beta)} = (X_{(\alpha,\beta)}, \xi_{(\alpha,\beta)})$ where:*

(1) $X_{(\alpha,\beta)} = X$

(2)

$\xi_{(\alpha,\beta)}) = \{B_{j,(\alpha,\beta)} \mid B_{j,(\alpha,\beta)} = \{x_i \in B_j \mid \mu^+(x_i) \geq \alpha, \mu^-(x_i) \leq \beta\}, i = 1,2,\ldots,n, j = 1,2,\ldots,k\}$

(3) $B_{k+1,(\alpha,\beta)} = \{x_i \notin B_j, i = 1,2,\ldots,n, j = 1,2,\ldots,k\}$

(α, β) - cut of bipolar fuzzy hypergraph is a a crisp hypergraph.

Example 4 $(0.5, -0.4)$ - cut of bipolar hypergraph in Figure 1 is shown in Figure 5. Here new edge $B_{6,(0.5,-0.4)}$ is added to contain the element x_1. The incidence matrix of $H_{(0.5,-0.4)}$ is as follows.

$$M_{(0.5,-0.4)} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{matrix} B_{1,(0.5,-0.4)} & B_{3,(0.5,-0.4)} & B_{4,(0.5,-0.4)} & B_{5,(0.5,-0.4)} & B_{6,(0.5,-0.4)} \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

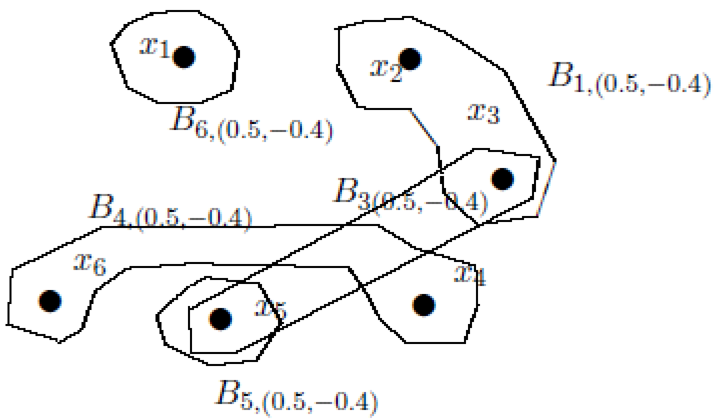


Figure 5: Example of cut level hypergraph of a bipolar fuzzy hypergraph.

If two sets of family of sets A, B be such that for each set A_1 of A there exists at least one set B_1 of B satisfying the relation $A_1 \subseteq B_1$. Symbolically it is written $A \hat{o} B$. If $A \hat{o} B$ and $A \neq B$, then $A \hat{\hat{o}} B$ is written.

Fundamental sequence of bipolar fuzzy hypergraph is an important term, defined below.

Definition 6 *Let* $H = (X, \xi)$ *be a bipolar fuzzy hypergraph, and for* $0 < \alpha \le h(H), d(H) \le \beta < 0$, *let* $h_{(\alpha,\beta)} = (X_{(\alpha,\beta)}, \xi_{(\alpha,\beta)})$ *be the* (α, β)*-level hypergraph of* H. *The sequence of real numbers* $\{s_k, s_{k-1}, \ldots, s_1, r_1, r_2, \ldots, r_n\}$ *such that* $d(H) = s_k < s_{k-1} < \ldots \le s_1 < 0 < r_1 < r_2 < \ldots < r_n = h(H)$ *which satisfies the following properties*

(1) If $s_{i+1} \le l < s_i, r_i < k \le r_{i+1}$, then $B_{(k,l)} = B_{(r_{i+1}, s_{i+1})}$,

(2) $B_{(r_{i+1}, s_{i+1})} \hat{\hat{o}} B_{(r_i, s_i)}$,

is called the fundamental sequence of H, and is denoted by $F(H)$.

For a hypergraph H, let fundamental sequence be $F(H)=\{s_k, s_{k-1},\ldots,s_1,r_1,r_2,\ldots,r_n\}$ where $k \le n$ be two positive integers. The core set of H is denoted by $C(H)$ and defined by $C(H)=\{H_{(r_1,s_1)}, H_{(r_2,s_2)},\ldots,H_{(r_k,s_k)}\}$.

We now define dual bipolar fuzzy hypergraph as follows.

Definition 7 *Let $H=(X,B)$ be a bipolar fuzzy hypergraph where $X=\{x_1,x_2,\ldots,x_n\}$ be a finite set and $B=\{B_1,B_2,\ldots,B_n\}$ be a bipolar fuzzy sets on subsets of X. The bipolar fuzzy hypergraph $\overline{H}=(\overline{B},\overline{X})$ is called dual bipolar fuzzy hypergraph of H if*

(1) $\overline{B}=\{b_1,b_2,\ldots,b_n\}$ is set of vertices of $\overline{H}$ corresponding to $B_1,B_2,\ldots,B_n$ respectively.

(2) $\overline{X}=\{\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_n\}$ where $\overline{x}_j=\{(b_j,\mu_j^+(b_j),\mu_j^-(b_j)) \mid \mu_i^+(b_j)=\mu_j^+(x_i), \mu_i^-(b_j)=\mu_j^-(x_i)$.

Example 5 The dual bipolar fuzzy hypergraph $\overline{H}$ of the hypergraph H defined in Figure 1 is shown in Figure 6. The corresponding incidence matrix M_D is evaluated.

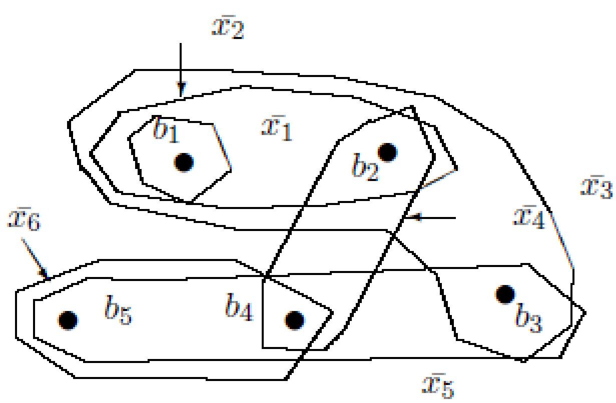


Figure 6: Dual bipolar fuzzy hypergraph of the bipolar fuzzy hypergraph.

$$M_D = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{array} \begin{array}{cccccc} \bar{x_1} & \bar{x_2} & \bar{x_3} & \bar{x_4} & \bar{x_5} & \bar{x_6} \end{array} \begin{pmatrix} (0.4,-0.3) & (0.5,-0.6) & (0.6,-0.4) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0.5,-0.3) & (0.4,-0.8) & (0.6,-0.2) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0.8,-0.4) & (0,0) & (0.9,-0.4) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0.5,-0.4) & (0.6,-0.2) & (0.9,-0.5) \\ (0,0) & (0,0) & (0,0) & (0,0) & (0.3,-0.5) & (0.5,-0.7) \end{pmatrix}$$

First we define elementary bipolar fuzzy set then elementary bipolar fuzzy hypergraph.

Definition 8 *A bipolar fuzzy set* $B = (\mu^+, \mu^-)$ *is called elementary bipolar fuzzy set if* $\mu^+ : X \to [0,1], \mu^- : X \to [-1,0]$ *are constant functions.*

Definition 9 *A bipolar fuzzy hypergraph is called elementary bipolar fuzzy hypergraph if all bipolar fuzzy edges are elementary.*

Definition 10 *Let* $H = (X, \xi)$ *be a bipolar fuzzy hypergraph and* $C(H) = \{H_{(r_1,s_1)}, H_{(r_2,s_2)}, \ldots, H_{(r_k,s_k)}\}$. H *is said to be ordered if* $C(H)$ *is ordered. The bipolar fuzzy hypergraph is simply ordered if* $C(H)$ *is simply ordered.*

Definition 11 *A bipolar fuzzy hypergraph* $H = (X, \xi)$ *is called* $\{m^+, m^-\}$ *tempered bipolar fuzzy hypergraph of a crisp hypergraph* $H^* = (X, E)$ *if there exists a bipolar fuzzy set* $B = (m^+, m^-)$ *such that* $\xi = \{(\gamma^+_{E_i}, \gamma^-_{E_i}) \mid E_i \in E\}$ *where*

$$\gamma^+_{E_i}(x) = \begin{cases} min\{m^+(e) \mid e \in E_i\} & if \;\; x \in E_i \\ 0, & if \;\; otherwise \end{cases}$$

and

$$\gamma^-_{E_i}(x) = \begin{cases} max\{m^-(e) \mid e \in E_i\} & if \;\; x \in E_i \\ 0, & if \;\; otherwise \end{cases}$$

Theorem 1 *A bipolar fuzzy hypergraph* $H = (X, \xi)$ *is a* $\{m^+, m^-\}$ *tempered bipolar fuzzy hypergraph of some crisp hypergraph* H^* *then* H *is elementary, support simple and simply ordered.*

Proof. Let $H = (X, \xi)$ is a $\{m^+, m^-\}$ tempered bipolar fuzzy hypergraph of some crisp hypergraph H^*. As it is $\{m^+, m^-\}$ tempered, the positive membership values and negative membership values of bipolar fuzzy edges of H are constant. Hence it is elementary. Clearly if support of two bipolar fuzzy edges of the $\{m^{+}, m^{-}\}$ tempered bipolar fuzzy hypergraph are equal then the bipolar fuzzy edges are equal. Hence it support simple. Let $C(H)=\{H_{(r_1,s_1)}, H_{(r_2,s_2)}, \ldots, H_{(r_k,s_k)}\}$ since H is elementary, it is ordered. Now we are to show that it is simple. Let $E \in H_{(r_{i+1},s_{i+1})} \setminus H_{(r_i,s_i)}$ then there exists $x^{*} \in E$ such that $\mu^{+}(x^{*}) = r_{i+1}$ and $\mu^{-}(x^{*}) = s_{i+1}$. Since $r_{i+1} < r_i, s_{i+1} > s_i$, it follows that $x^{*} \notin X_{(r_i,s_i)}$ and $E \not\subseteq X_{(r_i,s_i)}$. Hence H is simply ordered. □

Bipolar fuzzy transversal of bipolar fuzzy hypergraphs is defined below.

Definition 12 *Let $H = (X, \xi)$ be a bipolar fuzzy hypergraph. A bipolar fuzzy transversal $T = (\tau^{+}, \tau^{-})$ of H is a bipolar fuzzy set defined on X with the property that*

$T_{(h(B),d(B))} \cap B_{(h(B),d(B))} \neq \phi$ for each $B \in \xi$. A minimal bipolar fuzzy transversal T for H is a bipolar fuzzy transversal of H with the property that if $T_1 \subset T$, then T_1 is not a bipolar fuzzy transversal of H.

We denote set of minimal bipolar fuzzy transversal as $Tr(H)$. From the definition, it can be verified that $Tr(H) \neq \phi$.

Example 6 Let M_H be an incidence matrix of a bipolar fuzzy hypergraph H.

$$M_H = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{ccc} B_1 & B_2 & B_3 \\ \end{array}$$

$$M_H = \bordermatrix{ & B_1 & B_2 & B_3 \cr x_1 & (0.8,-0.7) & (0,0) & (0.6,0) \cr x_2 & (0.5,-0.6) & (0.5,-0.3) & (0.4,-0.5) \cr x_3 & (0.6,-0.4) & (0.4,-0.2) & (0.5,-0.4) }$$

Here $B_1 = \{(x_1, 0.8, -0.7), (x_2, 0.5, -0.6), (x_3, 0.6, -0.4)\}$ is a minimal bipolar fuzzy transversal.

3 Conclusions

Graph theory is an extremely useful tool in solving the combinatorial problems in different areas including geometry, algebra, number theory, topology, operations research, optimization and computer science. The bipolar fuzzy sets constitute a generalization of Zadeh's fuzzy set theory. The bipolar fuzzy models give more precision, flexibility and comparability to the system as compared to the classical and fuzzy models. We have introduced the concept of bipolar fuzzy hypergraphs, dual bipolar fuzzy hypergraphs, bipolar fuzzy transversal in this paper. The concept of the hypergraphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks and medical diagnosis.

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