APPROXIMATE CONTROLLABILITY RESULTS FOR IMPULSIVE LINEAR FUZZY STOCHASTIC DIFFERENTIAL EQUATIONS UNDER NONLOCAL CONDITIONS

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ABSTRACT

In this paper, the approximate controllability of impulsive linear fuzzy stochastic differential equations with nonlocal conditions in Banach space is studied. By using the Banach fixed point theorems, stochastic analysis, fuzzy process and fuzzy solution, some sufficient conditions are given for the approximate controllability of the system.

KEY WORDS

Approximate controllability, impulsive linear fuzzy stochastic differential equations, fuzzy solution, fixed point theorems, nonlocal conditions.

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1. INTRODUCTION

Differential equations have been used in modeling the dynamics of changing processes. A great dense of the modeling, development has been accompanied by a rich theory of differential equations. Freshly, stochastic differential equations (SDEs) occur in the mathematical modeling of various fields in physics and engineering science. Among them, several properties of SDEs such as existence, controllability, and stability are studied for the linear and nonlinear equations. But in many situations, it is useful to investigate the linear stochastic differential equations directly discussed by Daniel [2]. The properties and applications are presented in [5].

Fuzzy set theory introduced by Zadeh [19] is a generalization of abstract set theory. Semwal et. al [14] studied the less computationally intensive fuzzy logic based controller for humanoid push recovery. The theory of Impulsive differential equations have been developed in modeling impulsive problems in physics, population dynamics, ecology, biological systems, robotics, optimal control and so forth. In [6, 7, 9] these types of impulsive effects and differential systems were studied. The nonlocal condition, which is a generalization of the classical initial condition, was motivated by physical problems. Among others, we refer to the papers in [1,4,13]. In general, fixed point theorems are very useful classes of result that give us the conditions under which for some function f. The Sadovskii fixed point theorem and the theory of strongly continuous cosine families of operators are used to study the sufficient conditions for the controllability of the system considered and that are discussed in [3, 14,17].

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The concept of the controllability has played a central role during the history of modern control theory. The approximate controllable systems are more common and very often approximate controllability is completely sufficient in applications. Therefore, it is important and necessary to study the weaker concept of controllability, namely approximate controllability for linear and nonlinear differential systems. In [10,11] these types of controllability properties and conditions were discussed.

In the literature there are only some papers that deal with the approximate controllability of fuzzy differential and linear fuzzy differential systems, likewise approximate controllability of second order stochastic systems as discussed in [17]. Subalakshmi and Balachandran [16] studied the nonlinear stochastic impulsive integrodifferential system's approximate controllability. Mahmudov and Zorlu [8] discussed the concept of the approximate controllability of semilinear neural systems in Hilbert space. Narayanamoorthy and Sowmiya [12] presented the concept of the approximate controllability result for nonlinear impulsive neural fuzzy stochastic differential equation with nonlocal conditions. It should be mentioned that there is no work done on the approximate controllability of linear fuzzy stochastic differential equations. Motivated by the above consideration in this paper, we examine the approximate controllability for impulsive linear fuzzy stochastic differential equation with nonlocal conditions.

The rest of the paper is organized as follows. In the following section, we give the necessary preliminaries, definitions, lemmas and theorems. In section 3, we deduce the main result on the approximate controllability result for linear fuzzy stochastic differential systems. Finally, in Section 4 includes the conclusion.

2. PROBLEM FORMULATION AND PRELIMINARIES

Here, first we define some properties, theorems and lemma's and also recalls some basic definitions which are all used in this paper. A fuzzy set of R^n is a function u: $R^n \to [0, 1]$. For each fuzzy set u, we denote by $[u]^\alpha = x \in R^n$: $u(x) \geq \alpha$ for any $\alpha \in [0, 1]$, its $\alpha -$level set.

Let u, v be fuzzy sets of R^n. It is well known that $[u]^\alpha = [v]^\alpha$ for each $\alpha \in [0, 1]$ implies u =v. Let E^n denote the collection of all fuzzy sets of R^n that satisfies the following conditions:

- u is normal
- fuzzy convex
- upper semicontinuous and
- $[u]^0$ is compact.

We call $u \in E^n$ is an n-dimensional fuzzy number.

Definition 2.1. If $u \in E^n$, and $[u]^\alpha$ is a hyperrectangle, that is, $[u]^\alpha$ can be represented by $\prod_{i=1}^{n}[u_{il}^\alpha, u_{ir}^\alpha]$, that is $[u_{1l}^\alpha, u_{1r}^\alpha] \times [u_{2l}^\alpha, u_{2r}^\alpha] \times \ldots \times [u_{nl}^\alpha, u_{nr}^\alpha]$ for every $\alpha \in [0, 1]$, where $u_{il}^\alpha, u_{ir}^\alpha \in R$ with $u_{il}^\alpha \leq u_{ir}^\alpha$ when $\alpha \in (0, 1]$, i=1,2,...,n, then we call u is a fuzzy n-cell number. We denote the collection of all fuzzy n-cell numbers by $L(E^n)$.

Theorem 2.2.

For any u∈L(E^n) with $[u]^\alpha = \prod_{i=1}^{n}[u_{il}^{\alpha}, u_{ir}^{\alpha}]$($\alpha \in [0, 1]$), there exist a unique ($u_1,u_2,...,u_n$)∈(E^n) such that $[u]^\alpha = [u_{il}^{\alpha}, u_{ir}^{\alpha}]$(i=1,2,...,n and $\alpha \in [0, 1]$).

Conversely, for any ($u_1,u_2,...,u_n$)∈(E^n) with $[u]^\alpha = [u_{il}^{\alpha}, u_{ir}^{\alpha}]$(i=1,2,...,n and $\alpha \in [0, 1]$), there exit a unique u∈L(E^n) such that $[u]^\alpha = \prod_{i=1}^{n}[u_{il}^{\alpha}, u_{ir}^{\alpha}]$($\alpha \in [0, 1]$).

Definition 2.3. The complete metric D_L on $(E_N^i)^n$ is defined by

$$D_L(u,v) = \sup_{0<\alpha\le 1} d_L([u]^\alpha, [v]^\alpha) = \sup_{0<\alpha\le 1} \max_{1\le i\le n}\{|u_{il}^{\alpha} - v_{il}^{\alpha}|, |u_{ir}^{\alpha} - v_{ir}^{\alpha}|\} \tag{2.1}$$

for any u,v∈ L$(E_N^i)^n$, which satisfied $d_L(u+w, v+w) = d_L(u+v)$.

Definition 2.4. Let $u, v \in C([0,T]) : (E_N^i)^n)$, then

$$H_1(u,v) = \sup_{0\le t\le T} D_L(u(t), v(t)). \tag{2.2}$$

Definition: 2.5 [17]. A stochastic process x is said to be a mild solution of (3.1) if the following conditions are satisfied:

a. $x(t, \omega)$ is a measurable function from $J \times \Omega$ to H and $x(t)$ is Ft -adapted,
b. $E\|x(t)\|^2 < \infty$ for each $t \in J$,
c. $\Delta x(\tau_i) = x(\tau_i^+) - x(\tau_i^-) = I_i(x(\tau_i)). \qquad x \in X, 1 \le i \le m.$
d. For each $u \in L_2^F(J,U)$, the process x satisfies the following integral equation

$$\begin{aligned} x(t) &= U(t,0)[x^1 - \mu(x)] + \int_0^t U(t,s)Bu(s)ds \\ &+ \int_0^t U(t,s)f(s,x(s))ds + \int_0^t U(t,s)g(s,x(s))dW(s) \\ &+ \sum_{0\le t_i\le t} U(t,t_i)I_i(x(t_i^-)), \; t \in J. \end{aligned} \tag{2.3}$$

Definition: 2.6. Let $x, y \in C(I : E^n)$, here I be a real interval. A mapping $x : I \to E_N$ is called a fuzzy process. We denote,

$$[x(t)]^\alpha = [x_l^\alpha(t), x_r^\alpha(t)], \; t \in I, 0 < \alpha \le 1.$$

The derivative $x'(t)$ of a fuzzy process x is defined by

$$[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)], \; t \in I, 0 < \alpha \le 1$$

provided that is equation defines a fuzzy $x'(t) \in E_N$.

Definition: 2.7. The fuzzy process $x:[0,T]\to(E_N^i)^n$ with α-level set $[x(t)]^\alpha=\prod_{i=1}^{n}[x_{il}^\alpha,x_{ir}^\alpha]$ is a fuzzy solution of (3.1) with homogenous term iff

$$(x_{il}^a)'(t)=\min\{A_{il}^a(t)x_{ik}^a : j,k=l,r\}$$
$$(x_{ir}^a)'(t)=\min\{A_{ir}^a(t)x_{ik}^a : j,k=l,r\} \qquad (2.4)$$

and

$$x_{il}^a(0)+g_{il}^a(x_{il}^a)=x_{0_{il}}^a, x_{ir}^a(0)+g_{ir}^a(x_{ir}^a)=x_{0_{ir}}^a,\ i=1,2,...,n.$$

Lemma 2.8. Assume that $x\in B_h^{'}$, then for **t** ∈ **I**, x∈ B_h . Moreover,

$$l\,|\,x(t)\,|\le\|x_t\|_{Bh}\le\|x_0\|_{Bh}+l\sup_{s\in[0,t]}\|x(s)\|$$

where $l=\int_{-\infty}^{0}h(t)dt<\infty.$

Note 2.9. Define $B_h^{''}=z\in B_h^{'}:z_0=0\in B_h$. For any $z\in B_h^{''}$,

$$\|z\|_{Bh}^{''}=\|z_0\|_{Bh}+\sup|\,z(s)\,|:s\in[0,b]$$
$$=\sup|\,z(s)\,|:s\in[0,b],$$

and thus $(B_h^{''},\|\cdot\|)$ is a Banach space. Set

$$B_r=z\in PC((-\infty,b],X):\|z(t)\|_{Bh}^{'}\le r,\quad 0\le t\le b.$$

Clearly B_r is a nonempty, bounded, convex and closed set in $B_h^{'}$. Then for any z ∈ B_r, from Lemma [2.4], we have

$$\|z_t+y_t\|_{Bh}\le\|z_t\|_{Bh}+\|y_t\|_{Bh}$$

$$\le\|z_0\|_{Bh}+l\sup_{s\in[0,b]}|z(s)|+\|y_0\|_{Bh}+l\sup_{s\in[0,b]}|y(s)| \qquad (2.5)$$

$$\le l(r+M_1|\phi(0)|)+\|\phi\|_{Bh}=r^{'}$$

for each **t** ∈ **J**, z ∈ B_r, we have by above equation and (A_2)

$$\sup_{t\in J}|z(t)+y(t)|\le l^{-1}\|z_t+y_t\|_{Bh}\le l^{-1}r^{'}, \qquad (2.6)$$

$$|I_k(z(t_k^+)+z(t_k^-)|\le d_k(\sup_{t\in J}|z(t)+y(t)|)$$
$$\le d_k(l^{-1}r^{'}),\ k=1,2,...,m.$$

Let $x_b(x_0 ; u)$ be the state value of (3.1) at terminal time b corresponding to the control u and initial value $x_0 = \varphi \in B_h$. Introduce the set $R(b, x_0) = x_b(x_0 ; u)(0) : u(\cdot) \in L^2(J, U)$ which is called the reachable set at terminal time b, its closure in X is denoted by $\overline{R(b,\ x_0)}$.

Definition 2.10. If $\overline{R(b,\ x_0)} = X$ then the System (3.1) is approximately controllable on the interval J. It is convenient at this point to define operators

$$\Gamma_0^\alpha = \int_0^\alpha U(a,s)BB^* U^*(a,s)ds$$

$$R(\alpha,\Gamma_0^\alpha) = (\alpha I,\Gamma_0^\alpha)^{-1}$$

(D): $\alpha R(\alpha,\Gamma_{00}^\alpha) \to 0 \ as \ \alpha \to 0^+$ is strong operator topology. It is known that assumption (D) holds iff the linear system

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = x_0$$

is approximately controllable on J.

The following theorem gives a formula for a control transferring the initial state x_0 to some neighborhood of x_a at time a.

Theorem 2.11.

For arbitrary $x_a \in X$, the control

$$u(t) = B^* S^*(a-t) R(\alpha,\Gamma_0^\alpha)\, p(x(\cdot))$$

Where

$$p(x(\cdot)) = x_a - S(a)[x_0 - g(x)] - \int_0^a S(a-s)F(s)x(s)ds - \sum_{k=1}^n S(a-s)\int_0^a S(a-s)G_k(s)x(s)dW_k(s) - \sum_{j=1}^m S(a-t_j)(I_j)(x(t_j^-)) \tag{2.7}$$

$$x(a) = x_a - \alpha(\alpha I + \Gamma_0^a)^{-1}[S(a)[x_0 - g(x)] - \int_0^a S(a-s)F(s)x(s)ds - \sum_{k=1}^n S(a-s)\int_0^a S(a-s)G_k(s)x(s)dW_k(s) - \sum_{j=1}^m S(a-t_j)(I_j)(x(t_j^-))]. \tag{2.8}$$

3. MAIN RESULT

In this section, we verify that the approximate controllability result for impulsive linear fuzzy stochastic differential equation condition by using Banach fixed point theorem. Let us consider the impulsive linear fuzzy stochastic differential equation in $(E_N^i)^n$.

$$dx(t) = Ax(t) + F(t)x(t)dt + Bu(t)dt + \sum_{k=1}^{n} G_k(t)x(t)dW_k(t)$$
$$\Delta x(t_j) = x(t_j^+) - x(t_j^-) = I_j(x(t_j)), x \in X \qquad (3.1)$$
$$x(0) + g(x) = x_0 \in (E_N^i)^n$$

With fuzzy coefficient $A:[0,T] \to (E_N^i)^n$, $x_0 \in (E_N^i)^n$ is a initial value, and $u:[0,T] \to (E_N^i)^n$ is a control function.

To establish the result, we introduce the following assumptions on systems (3.1).

(A_1). S(t) is a fuzzy number satisfying for $y \in (E_N^i)^n$ $(d/dt)S(t)y \in C'(I:(E_N^i)^n$, $\cap C'(I:(E_N^i)^n)$ then,

$$(d/dt)S(t)y = A[S(t)y] = S(t)Ay, t \in I$$

where,

$$[S(t)]^\alpha = \prod_{i=1}^{n} [S_i(t)]^\alpha = \prod_{i=1}^{n} [S_{il}{}^\alpha(t), S_{ir}{}^\alpha(t)]$$

and $S_{ij}{}^\alpha(t)(j=l,r)$ is continuous with $|S_{ij}{}^\alpha(t)| \le C$, $C>0$, for all $t \in I = [0,T]$

(A2). The nonlinear fuzzy function $F:[0,T]\times(E_N^i)^n \to (E_N^i)^n$ and $g:(E_N^i)^n \to (E_N^i)^n$ is continuous function and both are satisfied the global Lipschitz condtions, such that,

$$d_L([Fx(s)]^\alpha, [Fy(s)]^\alpha) \le f d_L([x(s)]^\alpha, [y(s)]^\alpha)$$

and

$$d_L([gx(s)]^\alpha, [gy(s)]^\alpha) \le b d_L([x(.)]^\alpha, [y(.)]^\alpha)$$

for all $x(s), y(s), x(.), y(.) \in (E_N^i)^n$, f and b are a finite positive constants.

(A3) The fuzzy continuous function $G_k:[0,T]\times(E_N^i)^n \to (E_N^i)^n$ is a strongly measurable and satisfied the lipschitz condition, such that

$$d_L([G_k x(s)]^\alpha, [G_k y(s)]^\alpha) \le g d_L([x(s)]^\alpha, [y(s)]^\alpha)$$

for all $x(s), y(s) \in (E_N^i)^n$ and k=1, 2, … , n: g is positive constant.

(A4) The function $I_i : (E_N^i)^n \to (E_N^i)^n$ is a compact and there exist a positive constant d such that

$$d_L([I_i x(s)]^\alpha, [I_i y(s)]^\alpha) \le d d_L([x(.)]^\alpha, [y(.)]^\alpha)$$

for all $x(.), y(.) \in (E_N^i)^n$

(A5). $cb(1+T+cT)+fT(1+cT)+gT(1+cT)+d(1+cT)<1.$

From Note 2.6 and hypotheses (A1), equation (3.1) can be expressed as

$$x(t) = S(t)[x_0 - g(x)] + \int_0^t S(t-s)Bu(s)ds + \int_0^t S(t-s)Fx(s)ds + \sum_{k=1}^{n} \int_0^t S(t-s)G_k x(s)dW_k(s),$$

$$\Delta x(t_j) = x(t_j^+) - x(t_j^-) = I_j(x(t_j)),$$

$$x(0) + g(x) = x_0 \qquad (3.2)$$

Theorem 3.1

Let T > 0, if hypotheses (A1) – (A2) are hold, then for every $x_0 \in (E_N^i)^n$, equation (3.1)(u ≡ 0) have a unique fuzzy $x \in C([0,T]:(E_N^i)^n)$.

Theorem 3.2

Condition (A1) – (A4) and (D) are satisfied. Then the system (3.1) is approximate controllability on J.

Proof:

Let $\tilde{x}^\beta(\cdot)$ be a fixed point of Φ and any fixed point of Φ is a mild solution of (3.1) on [0, b]. By theorem 2.7, the control

$$\tilde{u}^\beta(t) = B^* S^*(a-t)\tilde{R}(\beta, \Gamma_o^a)[P(\tilde{x}^\beta)]^\alpha$$

which satisfies

$$[\tilde{x}^\beta(a)]^\alpha = [x_\alpha - \beta\tilde{R}(\beta,\Gamma_o^a)[x_\alpha - S_{il}(a)][x_0 - g_{il}(x)] - \int_0^a S_{il}^\alpha(a-s)F_{il}^\alpha(\tilde{x}^\beta)ds$$

$$-\sum_{k=1}^{n}\int_0^a S_{il}^\alpha(a-s)(G_k)_{il}^\alpha(\tilde{x}_s^\beta)dW_k(S) - \sum_{j=1}^{m} S_{il}^\alpha(a-t_j)(I_j)(x(t_j^-)),$$

$$x_\alpha - S_{il}^\alpha(a)[x_0 - g_{ir}(x)] - \int_0^a S_{ir}^\alpha(a-s)F_{ir}^\alpha(\tilde{x}^\beta)ds \qquad (3.3)$$

$$-\sum_{k=1}^{n}\int_0^a S_{ir}^\alpha(a-s)(G_k)_{ir}^\alpha(\tilde{x}_s^\beta)dW_k(S) - \sum_{j=1}^{m} S_{ir}^\alpha(a-t_j)(I_j)(x(t_j^-))]$$

where,

$$[P(\tilde{x}^{\beta})]^{\alpha} = [x_{\alpha} - S_{il}(a)][x_0 - g_{il}(x)] - \int_0^a S^{\alpha}_{il}(a-s)F^{\alpha}_{il}(\tilde{x}_s{}^{\beta})ds$$

$$-\sum_{k=1}^{n}\int_0^a S^{\alpha}_{il}(a-s)(G_k)^{\alpha}_{il}(\tilde{x}_s{}^{\beta})dW_k(S) - \sum_{j=1}^{m} S^{\alpha}_{il}(a-t_j)(I_j)(x(t_j^-)),$$

$$x_{\alpha} - S^{\alpha}_{ir}(a)[x_0 - g_{ir}(x)] - \int_0^a S^{\alpha}_{ir}(a-s)F^{\alpha}_{ir}(\tilde{x}^{\beta})ds$$

$$-\sum_{k=1}^{n}\int_0^a S^{\alpha}_{ir}(a-s)(G_k)^{\alpha}_{ir}(\tilde{x}_s{}^{\beta})dW_k(S) - \sum_{j=1}^{m} S^{\alpha}_{ir}(a-t_j)(I_j)(x(t_j^-))]$$

By (A2) $-$(A4)

$$d_L([F\tilde{x}_s{}^{\beta}]^{\alpha}, [F(s)]^{\alpha}) \le fd_L, d_L([(G_k)\tilde{x}_s{}^{\beta}]^{\alpha}, [(G_k)(s)]^{\alpha}) \le gd_L$$

$$d_L([g\tilde{x}_s{}^{\beta}]^{\alpha}, [g(s)]^{\alpha}) \le bd_L$$

Here, the sequence $F^{\alpha}_{il}(\tilde{x}_s{}^{\beta}), (G_k)^{\alpha}_{il}(\tilde{x}_s{}^{\beta})\, and\, g^{\alpha}_{il}(\tilde{x}_s{}^{\beta})$ are bounded in $(E^i_N)^n$. And $F^{\alpha}_{il}(s), (G_k)^{\alpha}_{il}(s)\, and\, g^{\alpha}_{il}(s)$ are all weakly convergent subsequence that are bounded in $(E^i_N)^n$, respectively. Take

$$\tilde{\omega} = [x_{\alpha} - S_{il}(a)][x_0 - g_{il}{}^{\alpha}(x)] - \int_0^a S^{\alpha}_{il}(a-s)F^{\alpha}_{il}(s)ds$$

$$-\sum_{k=1}^{n}\int_0^a S^{\alpha}_{il}(a-s)(G_k)^{\alpha}_{il}(s)dW_k(S) - \sum_{j=1}^{m} S^{\alpha}_{il}(a-t_j)(I_j)(x(t_j^-)),$$

$$x_{\alpha} - S^{\alpha}_{ir}(a)[x_0 - g_{ir}(x)] - \int_0^a S^{\alpha}_{ir}(a-s)F^{\alpha}_{ir}(s)ds$$

$$-\sum_{k=1}^{n}\int_0^a S^{\alpha}_{ir}(a-s)(G_k)^{\alpha}_{ir}(s)dW_k(S) - \sum_{j=1}^{m} S^{\alpha}_{ir}(a-t_j)(I_j)(x(t_j^-))].$$

It follows by the compactness of the operators

$q(.) \to \int_0^a U(.,s)q(s)ds:\ L^2(J,X) \to C(J,X),$ we obtain that

$$\| [P(\tilde{x}^{\gamma})]^{\alpha} - \tilde{\omega} \| \le \left\| \int_0^a S^{\alpha}_{ij}(a-s)[F^{\alpha}_{ij}(\tilde{x}_s{}^{\beta}), F^{\alpha}_{ij}(s)]ds \right\|$$

$$+ \left\| \sum_{k=1}^{n}\int_0^a S^{\alpha}_{il}(a-s)[(G_k)^{\alpha}_{il}(\tilde{x}_s{}^{\beta}), (G_k)^{\alpha}_{il}(s)]dW_k(S) \right\| \to 0\, as\, \beta \to 0^+$$

where, $j = l, r$. By Definition

$$\begin{aligned} \| [\tilde{x}^{\gamma}(a)]^{\alpha} - x_{\alpha} \| &= \left\| \beta \tilde{R}(\beta, \Gamma_{o}^{a})[P(\tilde{x}^{\beta})]^{\alpha} \right\| \\ &= \left\| \beta \tilde{R}(\beta, \Gamma_{o}^{a})([P(\tilde{x}^{\beta})]^{\alpha} - \tilde{\omega} + \tilde{\omega}) \right\| \\ &\leq \left\| \beta \tilde{R}(\beta, \Gamma_{o}^{a})\tilde{\omega} \right\| + \left\| \beta \tilde{R}(\beta, \Gamma_{o}^{a})[P(\tilde{x}^{\beta})]^{\alpha} - \tilde{\omega} \right\| \\ &\leq \left\| \beta \tilde{R}(\beta, \Gamma_{o}^{a})\tilde{\omega} \right\| + \left\| [P(\tilde{x}^{\beta})]^{\alpha} - \tilde{\omega} \right\| \to 0 \, as \, \beta \to 0^{+} \end{aligned}$$

ie., $\| [\tilde{x}^{\gamma}(a)]^{\alpha} - x_{\alpha} \| \to 0$. ie., this proves the approximate controllability of (3.1).

4. CONCLUSION

The article addresses the approximate controllability results for impulsive linear fuzzy stochastic differential equations with nonlocal conditions in Banach space. By using stochastic analysis, fuzzy process, fuzzy solution, Banach fixed point theorems and some sufficient conditions for the approximate controllability of the linear fuzzy stochastic control system are formulated and proved under the assumptions that are related linear system is approximate controllable.

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