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AN ARITHMETIC OPERATION ON HEXADECAGONAL FUZZY NUMBER

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ABSTRACT

In this paper, a new form of fuzzy number named as Hexadecagonal Fuzzy Number is introduced as it is not possible to restrict the membership function to any specific form. The $\alpha -$ cut of Hexadecagonal fuzzy number is defined and basic arithmetic operations are performed using interval arithmetic of $\alpha -$ cut and illustrated with numerical examples.

KEYWORDS

FUZZY NUMBERS, HEXADECAGONAL NUMBERS, ALPHA CUT, ARITHMETIC OPERATIONS

1. INTRODUCTION

L.A.Zadeh introduced fuzzy set theory in1965 [11]. Different types of fuzzy sets are defined in order to clear the vagueness of the existing problems. Membership function of these sets, which have the form A: R → [0, 1] and it has a quantitative meaning and viewed as fuzzy numbers. Hass. Michael [5], defines a fuzzy number as a quantity whose values are imprecise, rather than exact as in the case with single-valued function. So far, fuzzy numbers like triangular fuzzy numbers [3], trapezoidal fuzzy numbers [1], [10], hexagonal fuzzy numbers [8] are introduced with its membership functions. These numbers have got many applications like non-linear equations, risk analysis and reliability. Many operations were carried out using fuzzy numbers [4]. In this paper, we propose hexadecagonal fuzzy number with its membership functions and also we define basic arithmetic operations of hexadecagonal fuzzy number using arithmetic interval of alpha cuts and is illustrated with numerical examples.

2. PRELIMINARIES

2.1. Fuzzy set [11] :

A fuzzy set $\bar{A}$ in X (set of real numbers) is a set of ordered pairs $\bar{A}$ $=\{(x, \mu_{\bar{A}}(x))/x \in X\}\mu_{\bar{A}}(x)$ is called membership function of x in $\bar{A}$ which maps X into $[0,1]$.

2.2. Fuzzy Number [5] :

A fuzzy set $\bar{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number it its membership function has the following characteristics :

(i) $\bar{A}$ is convex i.e $\mu_{\bar{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\bar{A}}(x_1), \mu_{\bar{A}}(x_2))$ $\forall\, x_1, x_2 \in R$, $\lambda \in [0,1]$

(ii) $\bar{A}$ is normal i.e.,$\exists\, x_0 \in R$ such that $\mu_{\bar{A}}(x_0) = 1$

(iii) $\mu_{\bar{A}}$ is piecewise continuous.

2.3. Triangular Fuzzy Number [3] :

A fuzzy number $\bar{A}$= (a,b,c) is said to be a triangular fuzzy number if its membership function is given by,

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & for\ a \leq x \leq b \\ \dfrac{c-x}{c \quad b} & for\ b \leq x \leq c \\ 0 & otherwise \end{cases}$$

2.4. Trapezoidal Fuzzy Number [1] :

A fuzzy number $\bar{A}$ = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given by, where a ≤ b ≤ c ≤ d

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & for\ x < a \\ \dfrac{x-a}{b-a} & for\ a \leq x \leq b \\ 1 & for\ b \leq x \leq c \\ \dfrac{d-x}{d-c} & for\ c \leq x \leq d \\ 0 & for\ x > d \end{cases}$$

2.5. Hexagonal Fuzzy Number [8] :

A fuzzy number $\bar{A}_H$= $(a_1, a_2, a_3, a_4, a_5, a_6)$ is said to be hexagonal fuzzy number if its membership function is given by

$$\mu_{\bar{A}_H}(x) = \begin{cases} 0 & for\ \ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ \ a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \leq x \leq a_3 \\ 1 & for\ \ a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for\ a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \leq x \leq a_6 \\ 0 & for\ \ x > \ a_6 \end{cases}$$

2.6. α-cut of fuzzy set :

An $\alpha - cut$ of fuzzy set $\bar{A}$ is a crisp set defined as A_α={x$\in X / \mu_{\bar{A}}(x) \geq \alpha$}.

2.7. Convex fuzzy set:

A fuzzy set $\bar{A}$ is a convex fuzzy set if and only if each of its α-cut A_α is a convex set.

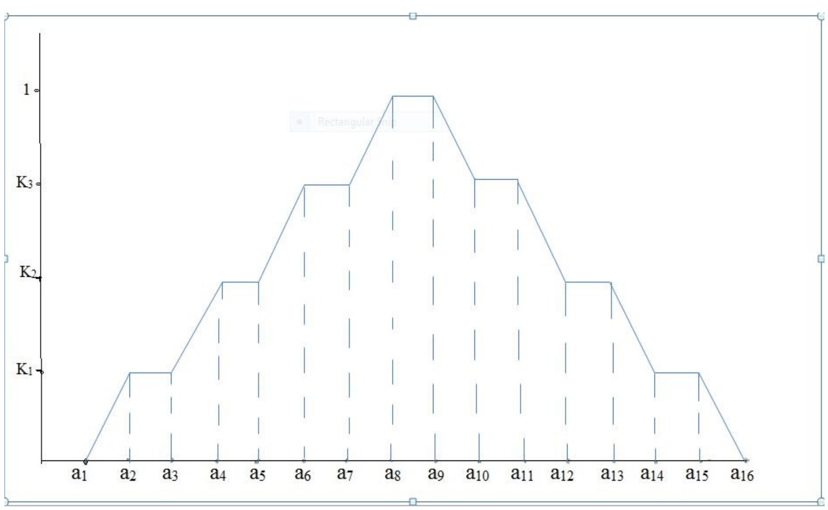
3. HEXADECAGONAL FUZZY NUMBER

In this section a new form of fuzzy number called Hexadecagonal fuzzy number is introduced which can be much useful in solving many decision making problems.

A fuzzy number $\bar{A}_{HD}= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ is said to be Hexadecagonal fuzzy number if its membership function is given by

$$\mu_{HD}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (k_2-k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ k_2 + (k_3-k_2)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ k_3 & a_6 \le x \le a_7 \\ k_3 + (1-k_3)\left(\frac{x-a_7}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 1 & a_8 \le x \le a_9 \\ k_3 + (1-k_3)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_3 & a_{10} \le x \le a_{11} \\ k_2 + (k_3-k_2)\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \le x \le a_{12} \\ k_2 & a_{12} \le x \le a_{13} \\ k_1 + (k_2-k_1)\left(\frac{a_{14}-x}{a_{14}-a_{13}}\right) & a_{13} \le x \le a_{14} \\ k_1 & a_{14} \le x \le a_{15} \\ k_1\left(\frac{a_{16}-x}{a_{16}-a_{15}}\right) & a_{15} \le x \le a_{15} \\ 0 & a_{16} \le x \end{cases}$$

Where $0 < k_1 < k_2 < k_3 < 1$.



3.1 Graphical representation of Hexadecagonal fuzzy number

3.2. Definition:

The parametric form of Hexadecagonal fuzzy number is defined as

$\bar{A} = \left(f_1(p),\ g_1(q), h_1(r), l_1(s), l_2(s), h_2(r), g_2(q), f_2(p)\right)$ for $p \in [0,k_1]$, $q \in [k_1,k_2]$, $r \in [k_2, k_3]$ & $s \in [k_3, w]$. $f_1(p),\ g_1(q), h_1(r), l_1(s)$ are bounded left continuous non decreasing functions over $[0,w_1]$ $[k_1,w_2]$ $[k_2,w_3]$ $[k_3,w_4]$ respectively, $f_2(p),\ g_2(q), h_2(r),\ l_2(s)$ are bounded left continuous non increasing functions over $[0,w_1]$ $[k_1,w_2]$ $[k_2,w_3]$ $[k_3,w_4]$ respectively, $0 \le w_1 \le k_1$, $k_1 \le w_2 \le k_2$, $k_2 \le w_3 \le k_3$ and $k_3 \le w_4 \le w$.

3.3. Arithmetic Operations on Hexadecagonal Fuzzy Number (HDFN):

3.3.1. Addition of two Hexadecagonal Fuzzy Numbers:

If $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ then

$$\bar{A}_{HD} + \bar{B}_{HD} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9, a_{10} + b_{10}, a_{11} + b_{11}, a_{12} + b_{12}, a_{13} + b_{13}, a_{14} + b_{14}, a_{15} + b_{15}, a_{16} + b_{16})$$

Example 3.1:

If $\bar{A}_{HD}$ = (1,2,3,5,6,8,9,10,11,13,15,16,17,18,19,20) and
$\bar{B}_{HD}$ = (1,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18) then
$\bar{A}_{HD} + \bar{B}_{HD} =$ (2,5,7,10,12,15,17,19,22,25,28,30,32,34,36,38)

3.3.2. Subtraction of two Hexadecagonal Fuzzy Numbers:

If $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ then

$$\bar{A}_{HD} - \bar{B}_{HD} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8, a_9 - b_9, a_{10} - b_{10}, a_{11} - b_{11}, a_{12} - b_{12}, a_{13} - b_{13}, a_{14} - b_{14}, a_{15} - b_{15}, a_{16} - b_{16})$$

Example 3.2:

If $\bar{A}_{HD}$ = (1,3,7,9,12,14,16,18,20,23,25,27,29,31,33,36) and
$\bar{B}_{HD}$ = (0,1,2,3,4,5,6,7,8,10,11,12,13,14,15,16) then
$\bar{A}_{HD} - \bar{B}_{HD} =$ (1,2,5,6,8,9,10,11,12,13,14,15,16,17,18,20)

3.3.3. Scalar Multiplication of two Hexadecagonal Fuzzy Numbers:

If $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
Then $k\ \bar{A}_{HD} =$
$(ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7, ka_8, ka_9, ka_{10}, ka_{11}, ka_{12}, ka_{13}, a_{14}, a_{15}, a_{16})$

Example 3.3

If $\bar{A}_{HD}$= (1,2,3,5,6,8,9,10,11,13,15,16,17,18,19,20)
2 $\bar{A}_{HD}$= (2,4,6,10,12,16,18,20,22,26,30,32,34,36,38,40)

3.3.4. Multiplication of two Hexadecagonal Fuzzy Numbers:

If $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ then

$$\bar{A}_{HD} * \bar{B}_{HD} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6, a_7 * b_7, a_8 * b_8, a_9 * b_9, a_{10} * b_{10}, a_{11} * b_{11}, a_{12} * b_{12}, a_{13} * b_{13}, a_{14} * b_{14}, a_{15} * b_{15}, a_{16} * b_{16})$$

Example 3.4:

If $\bar{A}_{HD}$= (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15) and
$\bar{B}_{HD}$= (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16) then

$$\bar{A}_{HD} * \bar{B}_{HD} = (0,2,6,12,20,30,42,56,72,90,110,132,156,182,210,240)$$

3.3.5. Equal Hexadecagonal Fuzzy Numbers :

Two Hexadecagonal Fuzzy Numbers

$\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$ are equal
i.e. $\bar{A}_{HD} = \bar{B}_{HD}$ iff $a_i = b_i \ \forall \ i$

3.3.6. Positive Hexadecagonal Fuzzy Number:

A positive Hexadecagonal Fuzzy Number (p-HDFN) is defined as
$p - \bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ where
$a_i > 0 \ \forall \ i.$

Example 3.5:

p- $\bar{A}_{HD}$= (1,2,3,5,6,8,9,10,11,13,15,16,17,18,19,20)

3.3.7. Negative Hexadecagonal Fuzzy Number:

A negative Hexadecagonal Fuzzy Number (n-HDFN) is defined as
$n - \bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ where
$a_i < 0 \ \forall \ i$

Example 3.6:

$n - \bar{A}_{HD}$=(-20,-19,-18,-17,-16,-15,-13,-11,-10,-9,-8,-6,-5,-3,-2,-1)

4. ALPHA CUT

4.1 Definition:

For $\propto \in [0,1]$, the $\alpha -$ cut of Hexadecagonal fuzzy number ,

$\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ is defined as $[\bar{A}_{HD}]_\alpha =$

$$\begin{array}{ll}
\left[a_1 + \frac{\alpha}{k_1}(a_2 - a_1), \quad a_{16} - \frac{\alpha}{k_1}(a_{16} - a_{15}) \right] & \text{for } \alpha \in [0, k_1] \\
\left[a_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_4 - a_3), \quad a_{14} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(a_{14} - a_{13}) \right] & \text{for } \alpha \in [k_1, k_2] \\
\left[a_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_6 - a_5), \quad a_{12} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(a_{12} - a_{11}) \right] & \text{for } \alpha \in [k_2, k_3] \\
\left[a_7 + \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_8 - a_7), \quad a_{10} - \left(\frac{\alpha - k_3}{1 - k_3}\right)(a_{10} - a_9) \right] & \text{for } \alpha \in [k_3, 1]
\end{array}$$

4.2 Operations of hexadecagonal fuzzy numbers using $\propto$- Cut:

The $\propto$ - Cut of hexadecagonal fuzzy number

$\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ for all $\propto \in [0,1]$ when $k_1 = \frac{1}{4}$, $k_2 = \frac{1}{2}$, $k_3 = \frac{3}{4}$ is given by

$$[\bar{A}_{HD}]_\alpha = \begin{cases}
[a_1 + 4 \propto (a_2 - a_1), a_{16} - 4 \propto (a_{16} - a_{15})] & for \ \propto \in [0.0.25] \\
[a_3 + (4 \propto -1)(a_4 - a_3), \ a_{14} - (4 \propto -1)(a_{14} - a_{13})] & for \ \propto \in [0.25.0.5] \\
[a_5 + (4 \propto -2)(a_6 - a_5), \ a_{12} - (4 \propto -2)(a_{12} - a_{11})] & for \ \propto \in [0.5, 0.75] \\
[a_7 + (4 \propto -3)(a_8 - a_7), \ a_{10} - (4 \propto -3)(a_{10} - a_9)] & for \ \propto \in [0.75, 1]
\end{cases}$$

4.2.1. Addition:

Let $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$
be two hexadecagonal fuzzy numbers . Let us add the alpha cuts of $[\bar{A}_{HD}]_\alpha$ and $[\bar{B}_{HD}]_\alpha$ of $\bar{A}_{HD}$ and $\bar{B}_{HD}$ using interval arithmetic.

$$[\bar{A}_{HD}]_\alpha + [\bar{B}_{HD}]_\alpha =$$

$$\begin{cases}
\begin{bmatrix} [a_1 + 4 \propto (a_2 - a_1), a_{16} - 4 \propto (a_{16} - a_{15})] + \\ [b_1 + 4 \propto (b_2 - b_1), b_{16} - 4 \propto (b_{16} - b_{15})] \end{bmatrix} & for \ \propto \in [0, 0.25] \\
\begin{bmatrix} [a_3 + (4 \propto -1)(a_4 - a_3), \ a_{14} - (4 \propto -1)(a_{14} - a_{13})] + \\ [b_3 + (4 \propto -1)(b_4 - b_3), \ b_{14} - (4 \propto -1)(b_{14} - b_{13})] \end{bmatrix} & for \ \propto \in [0.25, 0.5] \\
\begin{bmatrix} [a_5 + (4 \propto -2)(a_6 - a_5), \ a_{12} - (4 \propto -2)(a_{12} - a_{11})] + \\ [b_5 + (4 \propto -2)(b_6 - b_5), \ b_{12} - (4 \propto -2)(b_{12} - b_{11})] \end{bmatrix} & for \ \propto \in [0.5, 0.75] \\
\begin{bmatrix} [a_7 + (4 \propto -3)(a_8 - a_7), \ a_{10} - (4 \propto -3)(a_{10} - a_9)] + \\ [b_7 + (4 \propto -3)(b_8 - b_7), \ b_{10} - (4 \propto -3)(b_{10} - b_9)] \end{bmatrix} & for \ \propto \in [0.75, 1]
\end{cases}$$

Example 4.1:

If $\bar{A}_{HD}$ = (1,2,3,5,6,8,9,10,11,13,15,16,17,18,19,20) and
$\bar{B}_{HD}$= (1,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18)
For $\propto\in[0,0.25]$ $[\bar{A}_{HD}]_\alpha=[1+4\propto,20-4\propto]$ $[\bar{B}_{HD}]_\alpha=[1+8\propto,18-4\propto]$
$[\bar{A}_{HD}]_\alpha+[\bar{B}_{HD}]_\alpha=[2+12\propto,38-8\propto]$
When $\propto=0$ $[\bar{A}_{HD}]_0+[\bar{B}_{HD}]_0$=[2,38]
When $\propto=0.25$ $[\bar{A}_{HD}]_{0.25}+[\bar{B}_{HD}]_{0.25}$=[5,36]
For $\propto\in[0.25,0.5]$ $[\bar{A}_{HD}]_\alpha=[1+8\propto,19-4\propto]$ $[\bar{B}_{HD}]_\alpha=[3+4\propto,17-4\propto]$
$[\bar{A}_{HD}]_\alpha+[\bar{B}_{HD}]_\alpha=[4+12\propto,36-8\propto]$
When $\propto=0.25$ $[\bar{A}_{HD}]_{0.25}+[\bar{B}_{HD}]_{0.25}$=[7,34]
When $\propto=0.5$ $[\bar{A}_{HD}]_{0.5}+[\bar{B}_{HD}]_{0.5}$=[10,32]
For $\propto\in[0.5,0.75]$ $[\bar{A}_{HD}]_\alpha=[2+8\propto,18-4\propto]$ $[\bar{B}_{HD}]_\alpha=[4+4\propto,16-4\propto]$
$[\bar{A}_{HD}]_\alpha+[\bar{B}_{HD}]_\alpha=[6+12\propto,34-8\propto]$
When $\propto=0.5$ $[\bar{A}_{HD}]_{0.5}+[\bar{B}_{HD}]_{0.5}$=[12,30]
When $\propto=0.75$ $[\bar{A}_{HD}]_{0.75}+[\bar{B}_{HD}]_{0.75}$=[15,28]
For $\propto\in[0.75,1]$ $[\bar{A}_{HD}]_\alpha=[6+4\propto,19-8\propto]$ $[\bar{B}_{HD}]_\alpha=[5+4\propto,15-4\propto]$
$[\bar{A}_{HD}]_\alpha+[\bar{B}_{HD}]_\alpha=[11+8\propto,34-12\propto]$
When $\propto=0.75$ $[\bar{A}_{HD}]_{0.75}+[\bar{B}_{HD}]_{0.75}$=[17,25]
When $\propto=1$ $[\bar{A}_{HD}]_1+[\bar{B}_{HD}]_1$=[19,22]
Hence $\bar{A}_{HD}+\bar{B}_{HD}$ = (2,5,7,10,12,15,17,19,22,25,28,30,32,34,36,38)

4.2.2. Subtraction:

Let $\bar{A}_{HD}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16})$
$\bar{B}_{HD}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12},b_{13},b_{14},b_{15},b_{16})$

be two hexadecagonal fuzzy numbers . Let us subtract the alpha cuts of $[\bar{A}_{HD}]_\alpha$ and $[\bar{B}_{HD}]_\alpha$ of $\bar{A}_{HD}$ and $\bar{B}_{HD}$ using interval arithmetic.

$$[\bar{A}_{HD}]_\alpha-[\bar{B}_{HD}]_\alpha=$$

$$\begin{cases} \begin{bmatrix} [a_1+4\propto(a_2-a_1),a_{16}-4\propto(a_{16}-a_{15})]- \\ [b_1+4\propto(b_2-b_1),b_{16}-4\propto(b_{16}-b_{15})] \end{bmatrix} for \ \propto\in[0,0.25] \\ \begin{bmatrix} [a_3+(4\propto-1)(a_4-a_3),\ a_{14}-(4\propto-1)(a_{14}-a_{13})]- \\ [b_3+(4\propto-1)(b_4-b_3),\ b_{14}-(4\propto-1)(b_{14}-b_{13})] \end{bmatrix} for \ \propto\in[0.25,0.5] \\ \begin{bmatrix} [a_5+(4\propto-2)(a_6-a_5),\ a_{12}-(4\propto-2)(a_{12}-a_{11})]- \\ [b_5+(4\propto-2)(b_6-b_5),\ b_{12}-(4\propto-2)(b_{12}-b_{11})] \end{bmatrix} for \ \propto\in[0.5,0.75] \\ \begin{bmatrix} [a_7+(4\propto-3)(a_8-a_7),\ a_{10}-(4\propto-3)(a_{10}-a_9)]- \\ [b_7+(4\propto-3)(b_8-b_7),\ b_{10}-(4\propto-3)(b_{10}-b_9)] \end{bmatrix} for \ \propto\in[0.75,1] \end{cases}$$

Example 4.2

If $\bar{A}_{HD}$= (1,3,7,9,12,14,16,18,20,23,25,27,29,31,33,36) and
$\bar{B}_{HD}$= (0,1,2,3,4,5,6,7,8,10,11,12,13,14,15,16)
For $\propto\in[0,0.25]$ $[\bar{A}_{HD}]_\alpha=[1+8\propto,36-12\propto]$ $[\bar{B}_{HD}]_\alpha=[4\propto,16-4\propto]$
$[A_{HD}]_\alpha-[B_{HD}]_\alpha=[1+4\propto,20-8\propto]$

When $\propto = 0$ $[\bar{A}_{HD}]_0 - [\bar{B}_{HD}]_0 = [1,20]$
When $\propto = 0.25$ $[\bar{A}_{HD}]_{0.25} - [\bar{B}_{HD}]_{0.25} = [2,18]$
For $\propto \in [0.25,0.5]$ $[\bar{A}_{HD}]_\alpha = [5 + 8 \propto, 33 - 8 \propto]$ $[\bar{B}_{HD}]_\alpha = [1 + 4 \propto, 15 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha - [\bar{B}_{HD}]_\alpha = [4 + 4 \propto, 18 - 4 \propto]$
When $\propto = 0.25$ $[\bar{A}_{HD}]_{0.25} - [\bar{B}_{HD}]_{0.25} = [5,17]$
When $\propto = 0.5$ $[\bar{A}_{HD}]_{0.5} - [\bar{B}_{HD}]_{0.5} = [6,16]$
For $\propto \in [0.5,0.75]$ $[\bar{A}_{HD}]_\alpha = [8 + 8 \propto, 31 - 8 \propto]$ $[\bar{B}_{HD}]_\alpha = [2 + 4 \propto, 14 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha - [\bar{B}_{HD}]_\alpha = [6 + 4 \propto, 17 - 4 \propto]$
When $\propto = 0.5$ $[\bar{A}_{HD}]_{0.5} - [\bar{B}_{HD}]_{0.5} = [8,15]$
When $\propto = 0.75$ $[\bar{A}_{HD}]_{0.75} - [\bar{B}_{HD}]_{0.75} = [9,14]$
For $\propto \in [0.75,1]$ $[\bar{A}_{HD}]_\alpha = [10 + 8 \propto, 32 - 12 \propto]$ $[\bar{B}_{HD}]_\alpha = [3 + 4 \propto, 16 - 8 \propto]$
$[\bar{A}_{HD}]_\alpha - [\bar{B}_{HD}]_\alpha = [7 + 4 \propto, 16 - 4 \propto]$
When $\propto = 0.75$ $[\bar{A}_{HD}]_{0.75} - [\bar{B}_{HD}]_{0.75} = [10,13]$
When $\propto = 1$ $[\bar{A}_{HD}]_1 - [\bar{B}_{HD}]_1 = [11,12]$
Hence $\bar{A}_{HD} - \bar{B}_{HD}$ = (1,2,5,6,8,9,10,11,12,13,14,15,16,17,18,20)

4.2.3. Scalar Multiplication:

Let $A_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
be a hexadecagonal fuzzy number. Let us find the scalar multiplication of alpha cuts $[\bar{A}_{HD}]_\alpha$ of $\bar{A}_{HD}$ using interval arithmetic.

$$k\,[\bar{A}_{HD}]_\alpha = \begin{cases} [ka_1 + 4k \propto (a_2 - a_1), ka_{16} - 4k \propto (a_{16} - a_{15})] & for \propto \in [0,0.25] \\ [ka_3 + k(4 \propto -1)(a_4 - a_3),\ ka_{14} - k(4 \propto -1)(a_{14} - a_{13})] & for \propto \in [0.25,0.5] \\ [ka_5 + k(4 \propto -2)(a_6 - a_5),\ ka_{12} - k(4 \propto -2)(a_{12} - a_{11})] & for \propto \in [0.5,0.75] \\ [ka_7 + k(4 \propto -3)(a_8 - a_7),\ ka_{10} - k(4 \propto -3)(a_{10} - a_9)] & for \propto \in [0.75,1] \end{cases}$$

Example 4.3:

If $\bar{A}_{HD}$ = (1,2,3,5,6,8,9,10,11,13,15,16,17,18,19,20)
For $\propto \in [0,0.25]$ $[\bar{A}_{HD}]_\alpha = [1 + 4 \propto, 20 - 4 \propto]$ $2\,[\bar{A}_{HD}]_\alpha = [2 + 8 \propto, 40 - 8 \propto]$
$2\,[\bar{A}_{HD}]_0 = [2,40]$ $2\,[\bar{A}_{HD}]_{0.25} = [4,38]$
For $\propto \in [0.25,0.5]$ $[\bar{A}_{HD}]_\alpha = [1 + 8 \propto, 19 - 4 \propto]$ $2\,[\bar{A}_{HD}]_\alpha = [2 + 16 \propto, 38 - 8 \propto]$
$2\,[\bar{A}_{HD}]_{0.25} = [6,36]$ $2\,[\bar{A}_{HD}]_{0.5} = [10,34]$
For $\propto \in [0.5,0.75]$ $[\bar{A}_{HD}]_\alpha = [2 + 8 \propto, 18 - 4 \propto]$ $2\,[\bar{A}_{HD}]_\alpha = [4 + 16 \propto, 36 - 8 \propto]$
$2\,[\bar{A}_{HD}]_{0.5} = [12,32]$ $2\,[\bar{A}_{HD}]_{0.75} = [16,30]$
For $\propto \in [0.75,1]$ $[\bar{A}_{HD}]_\alpha = [6 + 4 \propto, 19 - 8 \propto]$ $2\,[\bar{A}_{HD}]_\alpha = [12 + 8 \propto, 38 - 16 \propto]$
$2\,[\bar{A}_{HD}]_{0.75} = [18,26]$ $2\,[\bar{A}_{HD}]_1 = [20,22]$
Hence $2\,\bar{A}_{HD}$= (2,4,6,10,12,16,18,20,22,26,30,32,34,36,38,40)

4.2.4. Multiplication:

Let $\bar{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$
$\bar{B}_{HD} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$
be two hexadecagonal fuzzy numbers. Let us multiply the alpha cuts of $[\bar{A}_{HD}]_\alpha$ and $[\bar{B}_{HD}]_\alpha$ of $\bar{A}_{HD}$ and $\bar{B}_{HD}$ using interval arithmetic.

$[\bar{A}_{HD}]_\alpha * [\bar{B}_{HD}]_\alpha =$

$$\begin{cases} \begin{bmatrix} [a_1 + 4 \propto (a_2 - a_1), a_{16} - 4 \propto (a_{16} - a_{15})] * \\ [b_1 + 4 \propto (b_2 - b_1), b_{16} - 4 \propto (b_{16} - b_{15})] \end{bmatrix} for \ \propto \in [0,0.25] \\ \begin{bmatrix} [a_3 + (4 \propto -1)(a_4 - a_3), \ a_{14} - (4 \propto -1)(a_{14} - a_{13})] * \\ [b_3 + (4 \propto -1)(b_4 - b_3), \ b_{14} - (4 \propto -1)(b_{14} - b_{13})] \end{bmatrix} for \ \propto \in [0.25,0.5] \\ \begin{bmatrix} [a_5 + (4 \propto -2)(a_6 - a_5), \ a_{12} - (4 \propto -2)(a_{12} - a_{11})] * \\ [b_5 + (4 \propto -2)(b_6 - b_5), \ b_{12} - (4 \propto -2)(b_{12} - b_{11})] \end{bmatrix} for \ \propto \in [0.5,0.75] \\ \begin{bmatrix} [a_7 + (4 \propto -3)(a_8 - a_7), \ a_{10} - (4 \propto -3)(a_{10} - a_9)] * \\ [b_7 + (4 \propto -3)(b_8 - b_7), \ b_{10} - (4 \propto -3)(b_{10} - b_9)] \end{bmatrix} for \ \propto \in [0.75,1] \end{cases}$$

Example 4.4:

If $\bar{A}_{HD}$= (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15) and
$\bar{B}_{HD}$= (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)
For $\propto \in [0,0.25]$ $[\bar{A}_{HD}]_\alpha = [4 \propto, 15 - 4 \propto]$ $[\bar{B}_{HD}]_\alpha = [1 + 4 \propto, 16 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha * [\bar{B}_{HD}]_\alpha = [4\alpha(1 + 4\alpha), (15 - 4\alpha)(16 - \alpha)]$ When
$\propto = 0$ $[\bar{A}_{HD}]_0 * [\bar{B}_{HD}]_0 = [0,240]$ When
$\propto = 0.25$ $[\bar{A}_{HD}]_{0.25} * [\bar{B}_{HD}]_{0.25} = [2,210]$
For $\propto \in [0.25,0.5]$ $[\bar{A}_{HD}]_\alpha = [1 + 4 \propto, 14 - 4 \propto]$ $[\bar{B}_{HD}]_\alpha = [2 + 4 \propto, 15 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha * [\bar{B}_{HD}]_\alpha = [(1 + 4 \propto)(2 + 4 \propto), (14 - 4 \propto)(15 - 4 \propto)]$
When $\propto = 0.25$ $[\bar{A}_{HD}]_{0.25} * [\bar{B}_{HD}]_{0.25} = [6,182]$
When $\propto = 0.5$ $[\bar{A}_{HD}]_{0.5} * [\bar{B}_{HD}]_{0.5} = [12,156]$
For $\propto \in [0.5,0.75]$ $[\bar{A}_{HD}]_\alpha - [2 + 4 \propto, 13 - 4 \propto]$ $[\bar{B}_{HD}]_\alpha - [3 + 4 \propto, 14 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha * [\bar{B}_{HD}]_\alpha = [(2 + 4 \propto)(3 + 4 \propto), (13 - 4 \propto)(14 - 4 \propto)]$
When $\propto = 0.5$ $[\bar{A}_{HD}]_{0.5} * [\bar{B}_{HD}]_{0.5} = [20,132]$
When $\propto = 0.75$ $[\bar{A}_{HD}]_{0.75} * [\bar{B}_{HD}]_{0.75} = [30,110]$
For $\propto \in [0.75,1]$ $[\bar{A}_{HD}]_\alpha = [3 + 4 \propto, 12 - 4 \propto]$ $[\bar{B}_{HD}]_\alpha = [4 + 4 \propto, 13 - 4 \propto]$
$[\bar{A}_{HD}]_\alpha * [\bar{B}_{HD}]_\alpha = [(3 + 4 \propto)(4 + 4 \propto), (12 - 4 \propto)(13 - 4 \propto)]$
When $\propto = 0.75$ $[\bar{A}_{HD}]_{0.75} * [\bar{B}_{HD}]_{0.75} = [42,90]$
When $\propto = 1$ $[\bar{A}_{HD}]_1 * [\bar{B}_{HD}]_1 = [56,72]$
Hence $\bar{A}_{HD} * \bar{B}_{HD} = (0,2,6,12,20,30,42,56,72,90,110,132,156,182,210,240)$

5. CONCLUSION

In this paper, a new form of fuzzy number named as Hexa-decagonal Fuzzy Number is introduced. The arithmetic operations are performed with arithmetic interval of alpha cuts and are illustrated with numerical examples. Hexa-decagonal Fuzzy Number can be applied to that problem which has sixteen points in representation. In future, it may be applied in operations research problems.

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