AN ALPHA -CUT OPERATION IN A TRANSPORTATION PROBLEM USING SYMMETRIC HEXAGONAL FUZZY NUMBERS

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***ABSTRACT*:**

In this paper we introduce a new operation on alpha cut for a symmetric hexagonal fuzzy numbers. We considered a transportation problem where the fuzzy demand and supply are in symmetric hexagonal fuzzy numbers and the minimum optimal cost is arrived .Transportation problems have various purposes in logistics and supply process for reducing the transportation cost's The advantages of the proposed alpha cut operations over existing methods is simpler and computationally more efficient in day to day applications.

***KEYWORDS*:**

Symmetric hexagonal fuzzy numbers, alpha cut, transportation problem, Robust's ranking,

1. INTRODUCTION:

The domains of number theories in mathematics have been continuously expanding from binary numbers (B), natural numbers (N), integers (Z), real numbers (R) to fuzzy numbers (F) and hyper structures (H).Fuzzy numbers and their fuzzy operations by Zadeh [14] are foundations of fuzzy number theory, fuzzy sets and fuzzy arithmetic for rigorously modeling fuzzy entities, phenomena, semantics, measurement, knowledge, intelligence, systems, and cognitive computational models. Fuzzy set theory was first proposed for decision making by Bellman and Zadeh [1]. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is described in the interval [0, 1]. It can be used in a wide range of domains where information is incomplete and imprecise. Dubois and Prade [2] in 1978 have defined any of the fuzzy numbers as a fuzzy subset of the real line. Since then, the application of fuzzy set theory to decision making in a fuzzy environment has been the issue of advanced research. In 2010, Pandian and Natarajan[9] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers. Nagoor Gani and Abdul Razak [8] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Also ranking of hexagonal fuzzy number plays a vital role in solving transportation problem [11]. Yong.D, Wenkang,S Feng.D and Qi.L proposed " A new similarity measure of generalized fuzzy numbers and its application to pattern recognition"[13]. Alpha cut method is standard method for performing different arithmetic operations like addition and subtraction. In this paper, we have defined arithmetic operations on symmetric hexagonal fuzzy numbers using α-cut.

2. PRELIMINARIES:

2.1. Fuzzy set [7]

A fuzzy set is characterized by a membership function mapping element of a domain, space or the universe of discourse X to the unit interval [0,1] (i.e.) $A = \{(x, \mu_A(x)); x \in X\}$. Here, $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ∈ X such that $\mu_A(x) = 1$.

2.3. Fuzzy number [5]

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following properties

i) A must be a normal fuzzy set.
ii) α_A must be a closed interval for every $\alpha \in (0,1]$.
iii) The support of A, ${}^{0+}A$, must be bounded.

2.4. Hexagonal Fuzzy number [10]

A fuzzy number $\bar{A}_H$ is a hexagonal fuzzy number denoted by $\bar{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6,)$ where a_1, a_2, a_3, a_4, a_5, a_6 are real numbers and its membership function $\mu_{\tilde{A}_H}$ is given as;

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right), & for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right), & for\ a_2 \le x \le a_3 \\ 1, & for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right), & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right), & for\ a_5 \le x \le a_6 \\ 0, & otherwise \end{cases}$$

2.5. Symmetric Triangular Fuzzy number [3]

If $a^{(2)}$=$a^{(3)}$, then the triangular fuzzy number A= ($a^{(1)}, a^{(2)}$, $a^{(3)}$) is called symmetric triangular fuzzy number. It is denoted by A = ($a^{(1)}, a^{(2)}$) , where $a^{(1)}$ is Core (A), $a^{(2)}$ is left width and right with of C.

2.6. Symmetric Hexagonal Fuzzy Number [12]

A symmetric hexagonal fuzzy number

$\tilde{A}_H = (a_L - s \ - t \ , a_L - s, a_L , \ a_U , \ a_U + s, a_U + s \ + t \ )$

Where, a_L, a_U , s and t are real numbers and its membership function is defined as

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(a_L-s-t)}{t}\right) & for\ a_L - s - t \le x \le a_L - s \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-(a_L-s)}{s}\right) & for\ a_2 - s \le x \le a_L \\ 1, & for\ \ a_L \le x \le a_U \\ 1-\frac{1}{2}\left(\frac{x-a_U}{s}\right), & for\ a_U \le x \le a_U + s \\ \frac{1}{2}\left(\frac{(a_U+s+t)-x}{t}\right), & for\ a_U + s \le x \le a_U + s + t \\ 0, & otherwise \end{cases}$$

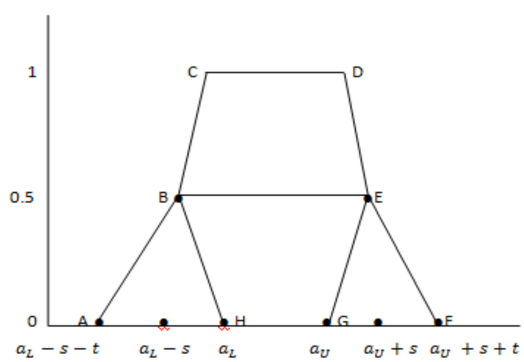

Figure 1. Symmetric Hexagonal Fuzzy Numbers

3. ALPHA CUT

3.1. Alpha cut [6]:

Given a fuzzy set A defined on X and any number $\alpha \in [0, 1]$ the α-cut, then we define α_A the crisp sets as $\alpha_A=\{x/A(x)\geq \alpha\}$

3.2. Strong alpha cut [6]:

Given a fuzzy set A defined on X and any number $\alpha \in[0,1]$ the strong α-cut then we define α_{+A} the crisp sets as $\alpha_{+A}= \{x/A(x)> \alpha\}$

3.3. Level set [4]:

The set of all levels $\alpha \in [0,1]$ that represents distinct α-cut of a given fuzzy set A is called a level set of A. Formally, $\wedge(A) = \{ \alpha / A(x) = \alpha$, for some $x \in X\}$ where, $\wedge$ denotes the level set of a fuzzy set A defined on X.

3.4. Arithmetic Operations of Symmetric Hexagonal Fuzzy numbers

Addition and Subtraction of two symmetric hexagonal fuzzy numbers can be performed as follows

Let $\tilde{A}_H = (a_L - s_1 - t_1, a_L - s_1, a_L, a_U, a_U + s_1, a_U + s_1 + t)$ and

$\tilde{B}_H = (b_L - s_2 - t_2, b_L - s_2, b_L, b_U, b_U + s_2, b_U + s_2 + t_2)$ be two symmetric hexagonal fuzzy numbers.

Addition:

$$\tilde{A}_H + \tilde{B}_H = (a_L + b_L) - t, (a_L + b_L) - s, (a_L + b_L), (a_U + b_U), (a_U + b_U + s), (a_U + b_U + t)$$

$$Where \quad s = (s_1 + s_2) \quad and \quad t = (s_1 + s_2 + t_1 + t_2)$$

Subtraction:

$$\tilde{A}_H - \tilde{B}_H = (a_L - b_U) - t, (a_L - b_U) - s, (a_L - b_L), (a_U - b_U), (a_U - b_L + s), (a_U - b_L + t)$$

$$Where \quad s = (s_1 + s_2) \quad and \quad t = (s_1 + s_2 + t_1 + t_2)$$

Example .3.4.1.

$$Let \quad \tilde{A}_H = (1,2,3,4,5,6) \quad and$$
$$\tilde{B}_H = (1,3,5,7,9,11)$$

be two symmetrical hexagonal fuzzy numbers

Then $\tilde{A}_H + \tilde{B}_H = (2,5,8,11,14,17) -------------- (1)$

The addition of two symmetrical hexagonal fuzzy numbers $\tilde{A}_H$ and $\tilde{B}_H$ is represented in Figure 2.

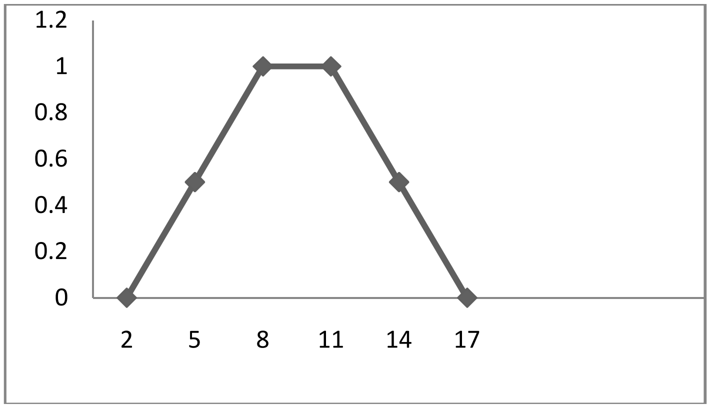

Figure2. Addition of Symmetric Hexagonal Fuzzy Numbers

Example.3.4.2.

$$Let \quad \tilde{A}_H = (1,4,6,10,12,15)$$
$$\tilde{B}_H = \quad (1,2,3,4,5,6)$$

be two symmetrical Hexagonal fuzzy numbers

Then $\tilde{A}_H - \tilde{B}_H = (-5,-1,3,6,10,14) ----------(2)$

The subtraction of two symmetrical hexagonal fuzzy numbers $\tilde{A}_H$ and $\tilde{B}_H$ is represented in Figure 3.

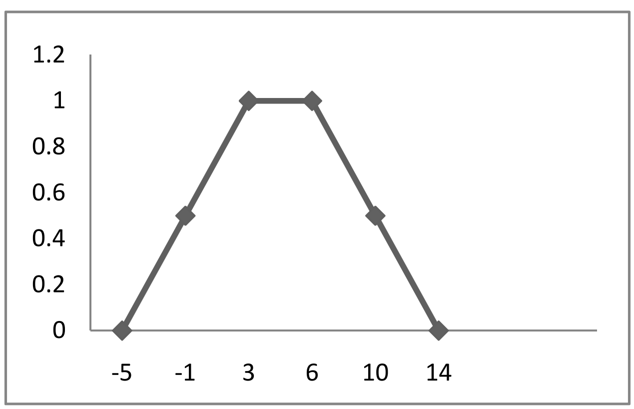

Figure3. Subtractions of Symmetric Hexagonal Fuzzy Numbers

3.5. Degree of membership function of alpha cut

The classical set $\tilde{A}_\alpha$ called alpha cut set of elements whose degree of membership is the set of elements in $\tilde{A}_H = (a_1, a_2, a_3, a_{4,} a_5, a_6)$ is not less than α .It is defined as

$$A_\alpha = \{x \in X / \mu_{\tilde{A}_H}(x) \geq \alpha \}$$
$$= \begin{cases} P_1(\alpha), P_2(\alpha) & for\, \alpha \in [0,0.5) \\ Q_1(\alpha), Q_2(\alpha) & for\, \alpha \in [0.5,1] \end{cases}$$

3.6. α Cut operations for the symmetrical hexagonal fuzzy numbers

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(a_L - s_1 - t_1)}{t_1}\right) & for\ a_L - s_1 - t_1 \leq x \leq a_L - s_1 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-(a_L - s_1)}{s_1}\right) & for\ a_L - s_1 \leq x \leq a_L \\ 1, & for\ a_L \leq x \leq a_U \\ 1 - \frac{1}{2}\left(\frac{x - a_U}{s_1}\right), & for\ a_U \leq x \leq a_U + s_1 \\ \frac{1}{2}\left(\frac{(a_U + s_1 + t_1) - x}{t_1}\right), & for\ a_U + s_1 \leq x \leq a_U + s_1 + t_1 \\ 0, & otherwise \end{cases}$$

and

$$\mu_{\tilde{B}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(b_L - s_2 - t_2)}{t_2}\right) & for\ b_L - s_2 - t_2 \leq x \leq b_L - s_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-(b_L - s_2)}{s_2}\right) & for\ b_L - s_2 \leq x \leq b_L \\ 1, & for\ b_L \leq x \leq b_U \\ 1 - \frac{1}{2}\left(\frac{x - b_U}{s_2}\right), & for\ b_U \leq x \leq b_U + s_2 \\ \frac{1}{2}\left(\frac{(b_U + s_2 + t_2) - x}{t_2}\right), & for\ b_U + s_2 \leq x \leq b_U + s_2 + t_2 \\ 0, & otherwise \end{cases}$$

As represented in Figure 4 an hexagonal fuzzy number denoted by $\tilde{A}_H$ /is defined as $\tilde{A}_H = (P_1(x), Q_1(x), Q_2(x_1), P_2(x_1))$ for x∈ [0, 0.5] and x_1∈ [0.5, 1] where,

(i) P_1 (x) is a bounded left continuous non decreasing function over [0, 0.5]

(ii) Q_1 (x) is a bounded left continuous non decreasing function over [0.5, 1]
(iii) Q_2 (x_1) is a bounded continuous non increasing function over [1, 0.5]
(iv) P_2 (x_1) is a bounded left continuous non increasing function over [0.5, 0]

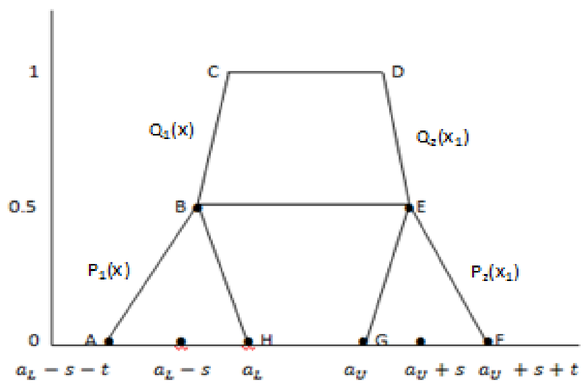

Figure 4 Graphical representation of Symmetrical hexagonal fuzzy number with x ∈ [0, 1]

Now for all x ∈ [0, 1] we can get crisp intervals by α cut operations

For all $\alpha \in [0,1]$ A_α shall be obtained as follows

Consider $P_1(x) \in [0,0.5]$

Let us assume that

$$P_1(x) = \alpha,$$
$$\frac{1}{2}\left(\frac{x-(a_L - s - t)}{t}\right) = \alpha \; for \; \alpha \in [0,0.5)$$
$$x = 2\alpha t + (a_L - s - t)$$
$$Hence \quad P_1(\alpha) = 2\alpha t + (a_L - s - t)$$

Similarly

Let us assume that

$$P_2(x_1) = \alpha,$$
$$\frac{1}{2}\left(\frac{(a_U + s + t) - x}{t}\right) = \alpha \; for \; \alpha \, \varepsilon (0.5,0]$$
$$x_1 = -2\alpha t + (a_U + s + t)$$
$$Hence \quad P_2(\alpha) = -2\alpha t + (a_U + s + t)$$

$$[P_1(\alpha), P_2(\alpha)] = [2\alpha t + (a_L - s - t), \quad -2\alpha t + (a_U + s + t)]$$

Next let us consider

$$Q_1(x) = \alpha,$$
$$\frac{1}{2} + \frac{1}{2}\left(\frac{x - (a_L - s)}{s}\right) = \alpha \ for\, \alpha \in (0.5,1]$$
$$x = 2\alpha s + a_L - 2s$$
$$Hence \quad Q_1(\alpha) = 2\alpha s + a_L - 2s$$

Similarly

$$Q_2(x_1) = \alpha,$$
$$1 - \frac{1}{2}\left(\frac{x - a_U}{s}\right) = \alpha \ for\alpha \in (1,0.5]$$
$$x_1 = -2\alpha s + a_U + 2s$$
$$Hence \quad Q_2(\alpha) = -2\alpha s + a_U + 2s$$

$$[Q_1(\alpha), Q_2(\alpha)] = [2\alpha s + (a_L - 2s), \quad -2\alpha s + (a_U + 2s)]$$

Hence
$$A_\alpha = \begin{cases} 2\alpha t_1 + (a_L - s_1 - t_1), \quad -2\alpha t_1 + (a_U + s_1 + t_1) & for\alpha \in [0,0.5) \\ 2\alpha s_1 + (a_L - 2s_1), \quad -2\alpha s_1 + (a_U + 2s_1) & for\alpha \in [0.5,1] \end{cases}$$

$$B_\alpha = \begin{cases} 2\alpha t_2 + (b_L - s_2 - t_2), \quad -2\alpha t_2 + (b_U + s_2 + t_2) & for\alpha \in [0,0.5) \\ 2\alpha s_2 + (b_L - 2s_2), \quad -2\alpha s_2 + (b_U + 2s_2) & for\alpha \in [0.5,1] \end{cases}$$

Hence, we can calculate the addition of fuzzy numbers using interval arithmetic is

$$A_\alpha + B_\alpha = \begin{cases} [2\alpha t_1 + (a_L - s_1 - t_1), -2\alpha t_1 + (a_U + s_1 + t_1)] \\ +[2\alpha t_2 + (b_L - s_2 - t_2), -2\alpha t_2 + (b_U + s_2 + t_2)] for\alpha \in [0,0.5) \\ \\ [2\alpha s_1 + (a_L - 2s_1), -2\alpha s_1 + (a_U + 2s_1)] \\ +[2\alpha s_2 + (b_L - 2s_2), -2\alpha s_2 + (b_U + 2s_2)] for\alpha \in [0.5,1] \end{cases},$$

Example3.6.1

Let $\tilde{A}_H = (1,2,3,4,5,6)$ and $\tilde{B}_H = (1,3,5,7,9,11)$ be two hexagonal fuzzy numbers .

$\tilde{A}_H + \tilde{B}_H = (2,5,8,11,14,17)$

For $\alpha \in [0,0.5)$

$A_\alpha = [2\alpha + 1, -2\alpha + 6]$ $B_\alpha = [4\alpha + 1, -4\alpha + 11]$

$A_\alpha + B_\alpha = [6\alpha + 2, -6\alpha + 17]$

For $\alpha \in [0.5,1]$

$A_\alpha = [2\alpha + 1, 2\alpha + 6]$ $B_\alpha = [4\alpha + 1, -4\alpha + 11]$

$A_\alpha + B_\alpha = [6\alpha + 2, -6\alpha + 17]$

Since for both $\alpha \varepsilon [0,0.5)$ and $\alpha \in [0.5,1]$ arithmetic intervals are same

Therefore $A_\alpha + B_\alpha = [6\alpha + 2, -6\alpha + 17]$ for all $\alpha \in [0,1]$

When $\alpha = 0$ $\quad \mathrm{A}_0 + \mathrm{B}_0 = [2,17]$

When $\alpha = 0.5$ $\quad \mathrm{A}_{0\cdot 5} + \mathrm{B}_{0\cdot 5} = [5,14]$

When $\alpha = 1$ $\quad \mathrm{A}_1 + \mathrm{B}_1 = [8,11]$

Hence, $\tilde{A}_H + \tilde{B}_H = (2,5,8,11,14,17) ---------(3)$

The above illustrations reveal that all the points coincide with the sum of the two symmetric hexagonal fuzzy numbers. Hence, the sum of the two α-cuts lies within the interval and the normal addition and $\alpha -$cut addition are equal (i.e.) (1) = (3)

3.7. Subtraction of two symmetric hexagonal fuzzy numbers

Let $\tilde{A}_H = (a_L - s_1 - t_1, a_L - s_1, a_L, a_U, a_U + s_1, a_U + s_1 + t_1)$ and

$\tilde{B}_H = (b_L - s_2 - t_2, b_L - s_2, b_L, b_U, b_U + s_2, b_U + s_2 + t_2)$

be two symmetric hexagonal fuzzy numbers for all $\alpha \in [0,1]$.Let us subtract the α cuts A_α *and* B_α *of* $\tilde{A}_H$ *and* $\tilde{B}_H$ using interval arithmetic.

$$\begin{aligned} A_\alpha - B_\alpha = [&2\alpha t_1 + (a_L - s_1 - t_1) - (-2\alpha t_2 + (b_U + s_2 + t_2)),\\ &2\alpha t_1 + (a_U + s_1 + t_1) - (2\alpha t_2 + b_L - s_2 - t_2),\\ &2\alpha(s_1) + (a_L - 2s_1 - (2\alpha s_2 + b_U + 2s_2),\\ &2\alpha s_1 + a_U + 2s_1 - (2s_2 + b_L - 2s_2)] \end{aligned}$$

Example 3.7.1

$\tilde{A}_H = (,2,4,6,10,12,14) \qquad \tilde{B}_H = (1,2,3,5,6,7)$
$\tilde{A}_H - \vec{B}_H = (-5,-2,1,7,10,13)$

$for, \alpha\varepsilon[0,0.5)$
$A_\alpha - B_\alpha = [4\alpha+2, \ -4\alpha+14]-[2\alpha+1, \quad -2\alpha+7]$
$=[6\alpha-5, \quad 6\alpha+13]$
$for \quad \alpha\varepsilon[0.5,1]$
$A_\alpha - B_\alpha = 4\alpha+2, -4\alpha+14]-[2\alpha+1, -2\alpha+7]$

$=[6\alpha-5, \ 6\alpha+13]$

Since for both $\alpha\varepsilon[0,0.5)$ and $\alpha \in [0.5,1]$ arithmetic intervals are same

For $\alpha = 0$ $A_0 - B_0 = [-5, 13]$
For $\alpha = 0.5$ $A_{0\cdot5} - B_{0\cdot5} = [-2, 10]$
For $\alpha = 1$ $A_1 - B_1 = [1, 7]$

Hence,

$A_\alpha - B_\alpha = [-5,-2,1,7,10,13]$ ------------ (4)

The above illustrations reveal that all the points coincide with the difference of the two symmetric hexagonal fuzzy numbers. Hence, the difference of the two α-cuts lies within the interval and the normal subtraction and α cut subtraction are equal (i.e) (2) = (4)

4. Transportation Problem With Symmetric Hexagonal Fuzzy Numbers

Let us consider a situation which has "m" origins (i.e.) supply points that has different types of goods

which has to be delivered to "n" demand points. We assume that the i^{th} origin must supply the fuzzy quantity $A_i = (a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}, a_i^{(5)}, a_i^{(6)})$ and at the same time the j^{th} destination must receive the fuzzy quantity $B_j = (b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}, b_j^{(5)}, b_j^{(6)})$

Let the fuzzy cost be $C_{ij} = (c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}, c_{ij}^{(5)}, c_{ij}^{(6)})$ for transporting a unit cost from the i^{th} origin to j^{th} destination. Our aim is to find the suitable origin and destination and number of units to be transported .We should analyze that all requirements are satisfied at a total minimum transportation cost for a balanced transportation problem.

The mathematical formulation of the symmetrical fuzzy transportation problem is as follows:

$$\text{Minimize} \quad \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \quad , \quad i = 1,2,3,.........,m ------(5)$$

$$\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{b}_j \quad , \quad j = 1,2,3,.........,n -------(6)$$

$$\sum_{i=1}^{n} \tilde{a}_i = \tilde{b}_j \quad , \quad i = 1,2,3,.........,m; \quad j = 1,2,3..........n$$

$$\text{and} \quad \tilde{x}_{ij} \geq 0 \quad \text{for all } i \text{ and } j.$$

It is important that, the transportation problem that we consider must be a linear programming problem (LPP). If there exist a feasible solution to the linear programming problem then it follows from (5) and (6) that,

$$\sum_{i=1}^{m}\sum_{j}^{n} \tilde{x}_{ij} = \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$

Also for the problem to be consistent the consistency equation is required.

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$

Where $\tilde{a}_{ij}$ is the fuzzy supply and $\tilde{b}_{ij}$ is the fuzzy demand

If the above equations holds we say that the linear programming problem is a balanced one where the supply and demand are equal

For an inconsistent equation we can see that

$$\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$$

The transportation model has a special structure which helps us to represent the linear programming problem in the form of a rectangular array commonly known as transportation table as given below.

S o u r c e s	Destination					
		1	2	…	N	Supply
	1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1n}$	$\tilde{a}_1$
	2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2n}$	$\tilde{a}_2$
	.	.	.	…	.	.
	M	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	…	$\tilde{c}_{mn}$	$\tilde{a}_m$
	Demand	$\tilde{b}_1$	$\tilde{b}_2$	…	$\tilde{b}_n$	

Table1. Transportation Table

4.1. Numerical Example

Consider the following fuzzy transportation problem. A company has three origins $O_{1,}O_2, O_3$ and four destinations $D_{1,}$ D_2 „D_3 and D_4. The fuzzy transportation cost for unit quantity of the product from i[th] source to j[th] destinations is C_{ij} where

$$\left[\tilde{C}_{ij}\right]_{3\times 4} = \begin{bmatrix} (1,2,3,4,5,6) & (1,3,5,6,8,10) & (9,11,13,16,18,20) & (2,4,6,8,10,12) \\ (1,3,5,7,8,10) & (0,2,4,6,8,10) & (2,3,4,5,6,7) & (3,6,9,10,13,16) \\ (3,5,7,8,10,12) & (4,8,12,16,20,24) & (3,4,5,6,7,8) & (6,8,10,11,13,15) \end{bmatrix}$$

The fuzzy production quantities per month at $O_{1,}O_2, O_3$ are (1,3,5,6,8,10), (2,4,6,7,9,11) and (3,6,9,17,20,23) tons respectively. The fuzzydemand per month for $D_{1,}$ $D_{2,}$ $D_{3,}$ and D_4 are (5,7,9,10,12,14), (1,3,5,6,8,10), (1,2,3,4,5,6) and (2,4,6,7,9,11) respectively.
The fuzzy transportation problem is,

$$\begin{array}{c} \\ O_1 \\ O_2 \\ O_3 \\ Demand \end{array} \begin{array}{ccccc} D_1 & D_2 & D_3 & D_4 & Supply \\ (1,2,3,4,5,6) & (1,3,5,6,8,10) & (9,11,13,16,18,20) & (2,4,6,8,10,12) & (1,3,5,6,7,10 \\ (1,3,5,7,8,10) & (0,2,4,6,8,10) & (2,3,4,5,6,7) & (3,6,9,10,13,16) & (2.4.6.7.9.11) \\ (3,5,7,8,10,12) & (4,8,12,16,20,24) & (3,4,5,6,7,8) & (6,8,10,11,13,15) & (3,6,9,17,20,23) \\ (5,7,9,10,12,14) & (1,3,5,6,8,10) & (1,2,3,4,5,6) & (2,4,6,7,9,11) & \end{array}$$

Solution:

Step 1: Construct the fuzzy transportation table for the given fuzzy transportation problem and then convert it into a balanced one, if it is not.

	Destination				
Origins	D_1	D_2	D_3	D_4	Supply
O_1	(1,2,3,4,5,6)	(1,3,5,6,8,10)	(9,11,13,16,18,20)	(2,4,6,8,10,12)	(1,3,5,6,7,10)
O_2	(1,3,5,7,8,10)	(0,2,4,6,8,10)	(2,3,4,5,6,7)	(3,6,9,10,13,16)	(2,4,6,7,9,11)
O_3	(3,5,7,8,10,12)	(4,8,12,16,20,24)	(3,4,5,6,7,8)	(6,8,10,11,13,15)	(3,6,9,17,20,23)
FD	(5,7,9,10,12,14)	(1,3,5,6,8,10)	(1,2,3,4,5,6)	(2,4,6,7,9,11)	

Table.2. Fuzzy Transportation Table

Step 2:

By using Robust's ranking method for symmetric hexagonal fuzzy numbers the transportation problem is further converted into crisp transportation problem.

If $\tilde{A}_H$ is a symmetric hexagonal fuzzy number then ranking of $\tilde{A}_H$ is given as

$$R(\tilde{A}_H) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha \text{ where}$$

$$(a_\alpha^{:L}, a_\alpha^U) = \begin{Bmatrix} (a_L - s - a_L + s + t)\alpha + a_L + s + t, & a_U - (a_U - a_L)\alpha \\ (a_U - a_L)\alpha + a_L & ,(a_U + s + t) - [(a_U + s + t) - (a_U + s)\alpha] \end{Bmatrix}$$

$$R(\tilde{A}_H) = \int_0^1 0.5\,[2(a_L + a_U)]d\alpha$$

$$\begin{bmatrix} R(1,2,3,4,5,6) & R(1,3,5,6,8,10) & R(9,11,13,16,18,20) & R(2,4,6,8,10,12) & R(1,3,5,6,7,10 \\ R(1,3,5,7,8,10) & R(0,2,4,6,8,10) & R(2,3,4,5,6,7) & R(3,6,9,10,13,16) & R\ (2.4.6.7.9.11) \\ R(3,5,7,8,10,12) & R(4,8,12,16,20,24) & R(3,4,5,6,7,8) & R(6,8,10,11,13,15) & R(3,6,9,17,20,23) \\ R(5,7,9,10,12,14) & R(1,3,5,6,8,10) & R(1,2,3,4,5,6) & R(2,4,6,7,9,11) & \end{bmatrix}$$

Applying Robust's ranking method the problem follows as

Now consider R (1, 2, 3, 4, 5, 6) $= \int_0^1 0.5\,[2(3+4)d\alpha = \int_0^1 0.5\,[14]d\alpha = \int_0^1 7d\alpha = 7$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs are calculated as:

R(1,3,5,6,8,10)=11 R(9,11,13,16,18,20)=29 R(2,4,6,8,10,12)=14 R(1,3,5,6,7,10)=11
R(1,3,5,7,8,10)=12 R(0,2,4,6,8,10)=10 R(2,3,4,5,6,7)=9 R(3,6,9,10,13,16)=19
R(2,4,6,7,9,11)=13 R(3,5,7,8,10,12)=15 R(4,8,12,16,20,24)=28 R(3,4,5,6,7,8)=11
R(6,8,10,11,13,15)=21 R(3,6,9,17,20,23)=26 R(5,7,9,10,12,14)=19 R(1,3,5,6,8,10)=11
R(1,2,3,4,5,6)=7 R(2,4,6,7,9,11)=13

	Destination				
Origin	D_1	D_2	D_3	D_4	Supply
O_1	7	11	29	14	11
O_2	12	10	9	19	13
O_3	15	28	11	21	26
Demand	19	11	7	13	

Table3. Crisp value of the transportation problem

The Initial basic feasible solution of the symmetric hexagonal Fuzzy transportation problem can be obtained by the Vogel's approximation method as follows. Now calculate the value difference for each row and column as mentioned in the last row and column the following table for fuzzy transportation problem is obtained.

	Destination					
Origin	D_1	D_2	D_3	D_4	Supply	R.D
O_1	7	11	29	**14**	11	4
O_2	12	10	9	19	13	1
O_3	15	28	11	21	26	4
Demand	19	11	7	**13**		
C.D	5	1	2	5 △		

Table 4. Allocation to the transportation problem

Identify the row /column corresponding to the highest value in difference. In this case it occurs in column 4.In this column minimum cost cell is (1, 4). The corresponding demand and supply are 13 and 11 respectively. Now allocate the maximum supply 11 to the minimum cost position at (1, 4). Remove the first row and repeat the above process to obtain the below table.

	Destination					
Origin	D_1	D_2	D_3	D_4	Supply	R.D
O_1	-	-	-	**11**	-	-
O_2	12	**10**	9	19	13	1
O_3	15	28	11	21	26	4
Demand	19	11	7	**2**		
C.D	3	18 △	2	2		

Table 5. Allocation to the transportation problem

The highest value of difference occurs in column 2.In this column minimum cost cell is (2, 2). The corresponding demand and supply are 11 and 13 respectively. Now allocate the maximum supply 13 to the minimum cost position at (2, 2). Remove the second column and repeat the same steps as above to obtain the below table.

	Destination					
Origin	D_1	D_2	D_3	D_4	Supply	R.D
O_1	-	-	-	**11**	-	-
O_2	12	**11**	-	19	2	7 △
O_3	15	-	**7**	21	19	6
Demand	19	-	-	**2**		
CD	3	-	-	2		

Table 6. Allocation to the transportation problem

The highest value in difference occurs in row 2.In this column minimum cost cell is (1, 2). The corresponding demand and supply are 19 and 2 respectively. Now allocate the maximum supply 2 to the minimum cost position at (1, 2). Remove the second row and repeat the process to obtain the below table.

	Destination					
Origin	D_1	D_2	D_3	D_4	Supply	R.D
O_1	-	-	-	**11**	-	-
O_2	**2**	**11**	-	-		
O_3	15	-	**7**	21	19	6
Demand	17	-	-	**2**		
CD						

Table7. Allocation to the transportation problem

The highest value in difference occurs in row3.In this row minimum cost cell is (3,1). The corresponding demand and supply are 17 and 19 respectively. Now allocate the maximum demand 17 to the minimum cost position at (3,1) and the remaining supply 2 units to cell (3,4) to get the optimal solution table.

	Destination					
Origin	D_1	D_2	D_3	D_4	Supply	R.D
O_1	-	-	-	**11**	-	-
O_2	**2**	**11**	-	-		
O_3	**17**	-	**7**	**2**		
Demand						
C.D		-	-			

Table 8. Allocation to the transportation problem

Step 3:

By solving the above interval linear programming problem we obtain the optimal solution. Therefore the symmetric hexagonal initial basic feasible solution in terms of symmetric hexagonal fuzzy numbers is given below.

FTP is $x_{14} = 11$ $x_{21} = 2$ $x_{22} = 11 x_{31} = 17 x_{33} = 7 x_{34} = 2$

The minimum total fuzzy transportation cost is given by

Minimize Z^I = (14 x 11) + (10 x 11) + (11 x 7) + (12 x 2) + (15 x 17) + (21 x 2) = Rs.662

5. CONCLUSION

In this paper, we have presented a feasible solution for a transportation problem using symmetric hexagonal fuzzy numbers which ensures the existence of an optimal solution to a balanced transportation problem. The shipping cost, availability at the origins and requirements at the destinations are all symmetrical hexagonal fuzzy numbers and the solution to the problem is given both as a fuzzy number and also as a ranked fuzzy number. This arithmetic operation of alpha cut provides more accurate results and gives us the minimum cost of transportation as compared to the earlier basic arithmetic operations in hexagonal fuzzy numbers. In few uncertain cases where there are six different points in which triangular and trapezoidal fuzzy numbers are not suitable symmetrical hexagonal fuzzy numbers can be used to solve such problems. Future work will represent generalized symmetric hexagonal fuzzy numbers in various fuzzy risk analysis and fuzzy optimization situations in an industry or any scientific research.

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