OWA BASED MAGDM TECHNIQUE IN EVALUATING DIAGNOSTIC LABORATORY UNDER FUZZY ENVIRONMENT

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ABSTRACT

The aim of this paper is to present an evaluation process using OWA operator in fuzzy Multi-attribute group decision making (MAGDM) technique for helping the health-care department to choose a suitable diagnostic laboratory among several alternatives. In the process of decision making, experts provide linguistic terms to evaluate each of the alternatives, which are parameterized by generalized triangular fuzzy numbers (GTFNs). Subsequently fuzzy MAGDM method is applied to determine the overall performance value for each alternative (laboratory) to make a final decision. Finally, the diagnostic laboratory evaluation problem is presented involving seven evaluation attributes, five laboratories and five experts.

KEYWORDS

Multi-attribute group decision making (MAGDM), Diagnostic laboratory, Approximate reasoning, Generalized fuzzy numbers, OWA operator

1. INTRODUCTION

In this era of advanced medical technology, health-care involves different disciplines and specialties. Prior to giving an opinion about the medical condition of a patient, a doctor most often requires additional information about the patient in the form of various diagnostic tests [1]. Diagnostic centers and laboratories are tools where these tests are conducted and the required information is acquired. With the increase of demand for improved and affordable services in health-care sector, a large number of diagnostic centers have come up. Each center claims itself on providing its patients with an optimum level of quality services. This gives rise to the need to assess and evaluate the performance of such centers and verify their claims, so that the patients can choose the best center and get a safe and pleasurable environment for their medical health check-ups. On the diagnostic center's perspective, the evaluation also helps to improve their efficiency and effectiveness [2].The performance evaluation and optimal selection of diagnostic centers have multilevel and multi-factor features. Thus the selection process can be regarded as multi-attribute decision making (MADM). MADM technique is the process of finding the best option from a finite set of available alternatives characterized by multiple, potentially conflicting attributes [3]. In MADM situations, linguistic variables are used for modeling human judgments due to subjective estimation and perception, incomplete knowledge, thecomplexity of studied systems [4,5]. These issues were not easily solved until the development of Zadeh's fuzzy sets theory [6]. There are several studies which are related to the performance evaluation under fuzzy environment [7-14].

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The main objectives of this study are to propose a systematic evaluation model to help the health-care department selecting an optimal diagnostic laboratory among a set of available alternatives under fuzzy multi-attribute environment. Basically, the evaluation has been done from the patients' perspectives. Hence, this study utilizes a group decision making (GDM) technique [15] to obtain the performance ratings of the alternatives in linguistic terms parameterized with generalized triangular fuzzy numbers (GTFNs).

The organization of the paper is as follows: Section 2 briefly introduces the theory of generalized fuzzy numbers (GFNs) and Ordered Weighted Aggregation (OWA) oprator. A stepwise framework to evaluate a diagnostic laboratory using a fuzzy GDM technique is derived in Section 3. This section alsopresents an empirical diagnostic laboratory selection problem. Finally conclusions and suggestions are presented in Section 4.

2. PRELIMINARIES

2.1. GFNs Representing Experts' Opinions

In any real-life decision making situation an expert may feel comfortable to express his/her opinions in natural languages. These linguistic opinions are then suitably quantified by GFNs since it considers the degrees of confidence of expert's opinions. In the year 1985 and 1999 Chen introduced the concepts of GFNs [16,17] following the study of normalized fuzzy numbers.

A GFN $\tilde{A} = (m_1, m_2;\ \beta, \gamma, w)$ is a fuzzy subset on the real line **R**, where $0 < w \leq 1$, and m_1, m_2, β and γ are real numbers. The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ satisfies the following conditions:

(*i*)) $\mu_{\tilde{A}}$ is a continuous mapping from **R**to the closed interval$[0, w]$, $0 < w \leq 1$;
(ii) $\mu_{\tilde{A}} = 0$, where $-\infty < x \leq m_1 - \beta$;
(iii)) $\mu_{\tilde{A}}$ is strictly increasing on $[m_1 - \beta, m_1]$;
(iv)) $\mu_{\tilde{A}} = w$, where $m_1 < x \leq m_2$;
(v)) $\mu_{\tilde{A}}$ is strictly decreasing on $[m_2, m_2 + \gamma]$;
(vi)$\mu_{\tilde{A}} = 0$, where $m_2 + \gamma \leq x < \infty$;

If $\mu_{\tilde{A}}$ is a linear function of x, $\tilde{A}$ is called a generalized trapezoidal fuzzy number.
If $\mu_{\tilde{A}}$ is a linear function of x and also $m_1 = m_2 = m$ then $\tilde{A}$ is called a GTFN.
If $w = 1$, then the GFN $\tilde{A}$ is called a normalized fuzzy number and denoted as $\tilde{A} = (m_1, m_2;\ \beta, \gamma)$. If $\beta = 0$ and $\gamma = 0$, then $\tilde{A}$ is called a crisp interval.
If$\beta = 0, \gamma = 0, m_1 = m_2$, and $w = 1$, then $\tilde{A}$ is called a real number.

If a GFN$\tilde{A}$ denotes an expert's opinion, then the value w denotes the corresponding degree of confidence of the expert. Without any loss of generality throughout the paper, to represent the linguistic assessments provided by the experts in a flexible way, we have considered GTFN of the form $\tilde{A} = (m;\ \beta, \gamma, w)$ with m, β and γ are positive real numbers and $0 < w \leq 1$.

2.2. Ordered Weighted Aggregation (OWA) Operator [31]

The Ordered Weighted Aggregation (OWA) operatorhas a wide range of applicability in group decision making where a fix level priority is given to different decision maker to obtain an optimistic or pessimistic decision. OWA is a function $F: R^n \to R$ which is defined as follows:

$$F(a_1, a_2, \dots a_n) = \omega_1 b_1 + \omega_2 b_2 + \cdots + \omega_n b_n.$$

Where, b_i is the i^{th} largest element of $(a_1, a_2, \dots a_n)$ and $(\omega_1, \omega_2, \dots \omega_n)$ is thecorresponding weight vector. Several proposals on weights determination of OWA operator are present in literature among which mathematical programming based methods are attracting a large number of researches [30]. Based on simplicity, the method proposed by Fullér and Majlender [30] is chosen for this study. This method obtains the weight vector $\omega = (\omega_1, \omega_2, \dots \omega_n)$ by minimizing the *variability* of the weights for a given

$$orness(\omega) = \alpha = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\omega_i \text{ (measure of optimism).}$$

The *variability* of the weights is denoted as $D^2(\omega)$ and defined as

$$D^2(\omega) = \frac{1}{n}\sum_{i=1}^{n}\left(\omega_i - \frac{1}{n}\right)^2.$$

The analytical solution of this method is also given as:

$$\omega_i = \frac{n-i}{n-1}\omega_1 + \frac{i-1}{n-1}\omega_n \text{for } (i = 2,3 \dots n-1),$$

where ω_1 and ω_n are given as follows

$$\omega_1 = \frac{2(2n-1) - 6(n-1)(1-\alpha)}{n(n+1)} \text{ and } \omega_n = \frac{6(n-1)(1-\alpha) - 2(n-2)}{n(n+1)}.$$

3. FRAMEWORK FOR DIAGNOSTIC LABORATORY EVALUATION

A structure for tackling the problems of evaluating the diagnostic laboratory in a fuzzy environment is constructed in this section. The content consists of three subsections: investigating the evaluation attributes, applying GDM technique to obtain the overall performance value of each laboratory and finally describing a ranking process. The structure of GDM is given below.

Given n experts $\{J_1, J_2, \dots J_n\}$, m alternatives $\{A_1, A_2, \dots A_m\}$,and p attributes $C = \{C_1, C_2, \dots C_p\}$,a typical fuzzy multi-attribute group decision-making (MAGDM) problem can be expressed in matrix format as follows:

$$\tilde{D}_i = \begin{matrix} & A_1 & A_2 & \dots & A_m \\ C_1 & \tilde{a}_{i11} & \tilde{a}_{i12} & \cdots & \tilde{a}_{i1m} \\ C_2 & \tilde{a}_{i21} & \tilde{a}_{i22} & \dots & \tilde{a}_{i2m} \\ \vdots & \vdots & \vdots & & \vdots \\ C_p & \tilde{a}_{ip1} & \tilde{a}_{ip2} & \cdots & \tilde{a}_{ipm} \end{matrix}$$

Where$\tilde{a}_{ijk} = (m_{ijk}; \beta_{ijk}, \gamma_{ijk}; w_{ijk})$ be the linguistic assessment provided by the expert $J_i (i = 1,2, \dots n)$, for every alternative $A_k (k = 1,2, \dots m)$ depending on the attribute $C_j (j = 1,2 \dots p)$. Here w_{ijk} denotes the corresponding degree of confidence of the expert's opinions.

3.1. Investigating the Evaluation Attributes

A five experts committee is formed, by the health care department, to choose the best diagnostic laboratory of a certain locality. In this evaluation process a questionnaire is provided to get the final evaluations. In order to achieve the goal, the experts move to different diagnostic laboratories and notice the condition, cleanliness of the laboratories. They used to talk with the technicians; doctors, examine different reagent [18]and also monitor the day-to-day experiences (findings) of patients. Finally, they make their comments in the questionnaire. The questionnaire containing seven attributes that affect the decisions of the experts is given below.

Questionnaire:

C_1: Behavior at the reception registration
- How the information and guidance are given to the client?
- Is there any priority in registration and processing?

C_2: Technical know-how
- What is your opinion about the technicians' helpfulness and skill?
- Are the medical practitioners enough qualified and experienced?

C_3: Cleanliness and Ambience
- Are you satisfied with the cleanliness and ambience of the center?

C_4: About service
- Are you getting complete health diagnosis under one roof?
- Does the center provide best quality reagents and equipments?
- Are disposable operators used to collect the blood sample?
- What is your opinion about the accuracy and reliability of the center?
- Are you getting personalized professional attention?
- Is international standard of hygiene being provided to you?

C_5: Interaction
- If you are more comfortable in a particular language, is the laboratory helping you by providing technicians who know the required language?
- Are the clients satisfied with their interaction with the doctors?

C_6: Is the program cost effective?

C_7: Is there any delay of service?

These attributes are classified into two groups, i.e., benefit and cost attributes. The experts' intentions are to give the maximum rating to the alternative with maximum benefit and minimum cost. In this respect, attributes C_1 to C_5 are classified as benefit attributes and $C^{/} = \{C_1, C_2, C_3, C_4, C_5\}$.

Moreover, the experts' challenge is also to identify the centre that provides services with affordable/lesser costs. Again according to the experts, delay in diagnosis may cause serious problems. Thus, the cost attributes areC_6,C_7 and $C^{//} = \{C_6, C_7\}$.

3.2. Fuzzy GDM Technique in Deriving the Overall Performance Values of Alternatives

In any decision support system it is very rare when all individuals in a group share the same opinion about the alternatives, since a diversity of backgrounds commonly exists. Hence, finding a group consensus to represent a common opinion of the experts is an important research topic of GDM. In view of this, several aggregation methods have been proposed to combine the individual opinions on GDM under fuzzy environment [19-24]. OWA opeartor facilitates to accomodate decision maker's attitude ranging from pessimistic to optimistic. Keeping in mind the fact that the experts' confidence levels affect the decision, the aggregation process developed in the paper of

Guha and Chakraborty is employed here to obtain each alternative's overall performance [15]. The aggregation process is described as follows.

- ✓ Construction of fuzzy decision matrix $D_i = (\tilde{a}_{ijk})_{p\times m}\ (i = 1,2, \dots n)$ for individual expert J_i.
- ✓ Normalization of each fuzzy decision matrix of individual expert $J_i N_i = (\tilde{r}_{ijk})_{p\times m} (i = 1,2, \dots n)$.
- ✓ Construction of aggregated matrix of first kind after accumulating all n experts' opinions regarding individual alternative.
- ✓ Construction of aggregated matrix of second kind after overall assessments of an alternative against each expert.
- ✓ Computation of average agreement degree of each expert for each alternative using OWA operator.

3.2.1. Normalization of each fuzzy decision matrix

Since different types of information are measured in different scales or in other word since the attributes are incommensurable we have to normalize the decision matrices $\widetilde{D}_i = \left(\tilde{a}_{ijk}\right)_{p\times m} (i = 1,2, \dots n)$ into corresponding decision matrices $\widetilde{N}_i = \left(\tilde{r}_{ijk}\right)_{p\times m} (i = 1,2, \dots n)$, where $\tilde{r}_{ijk}$ takes the following form:

$$\tilde{r}_{ijk} = \left(\frac{m_{ijk}}{d_j^{max}}; \frac{\beta_{ijk}}{d_j^{max}}, \frac{\gamma_{ijk}}{d_j^{max}}; w_{ijk}\right) \quad for \quad C_j \in C^{/}$$

$$\text{and}\tilde{r}_{ijk} = \begin{cases} \left(\frac{a_j^{min}}{m_{ijk}}; \frac{a_j^{min}.\gamma_{ijk}}{m_{ijk}(m_{ijk}+\gamma_{ijk})}, \frac{a_j^{min}.\beta_{ijk}}{m_{ijk}(m_{ijk}-\beta_{ijk})}; w_{ijk}\right) \left(a_j^{min} \neq 0\right) \\ \left(1 - \frac{m_{ijk}}{d_j^{max}}; \frac{\beta_{ijk}}{d_j^{max}}, \frac{\gamma_{ijk}}{d_j^{max}}; w_{ijk}\right) \qquad (a_j^{min} = 0) \end{cases} \quad for \quad C_j \in C^{//}$$

Where
$$d_j^{max} = \max_{1\le k\le m}\{m_{ijk} + \gamma_{ijk} | \tilde{a}_{ijk} = (m_{ijk}; \beta_{ijk}, \gamma_{ijk}; w_{ijk})\} \ \ and$$
$$a_j^{min} = \min_{1\le k\le m}\{m_{ijk} - \beta_{ijk} | \tilde{a}_{ijk} = (m_{ijk}; \beta_{ijk}, \gamma_{ijk}; w_{ijk})\}$$

3.2.2. Construction of aggregated matrix of first kind after accumulating all n experts' opinions regarding individual alternative

In order to accumulate experts opinion collectively against each alternative in this step decision makers' level of confidence are incorporated. In this regard the aggregated matrix of first kind $\widetilde{N} = (\tilde{r}_{jk})_{p\times m}$ has been constructed from the individual normalized fuzzy decision matrices $\widetilde{N}_i = (\tilde{r}_{ijk})_{p\times m}$ (i=1,2...n), where the aggregated fuzzy assessments are $\tilde{r}_{jk} = \sum_{i=1}^{n} \tilde{r}_{ijk}$. The degrees of confidence of experts regarding their evaluation $\tilde{r}_{jk}$ may be defined as $w_{jk} = \min_{1\le i\le n} w_{ijk}$.

3.2.3. Construction of aggregated matrix of second kind after overall assessments of an alternative against each expert

Here expert's linguistic cardinal priority is included in the decision process. Say, $\tilde{P}_j^i$s are the importance given by i^{th} expert for j^{th} attribute. For this reason, for each alternative A_k, let $\tilde{u}_{ik}$ be the assessments over all attribute measures are calculated (see Table 5).by each expert, which may be defined as follows:

$$u_{ik} = \sum_{i=1}^{p} \tilde{P}_j^i \,(\times) \tilde{r}_{ijk} / \sum_{i=1}^{p} \tilde{P}_j^i \qquad (1)$$

The above evaluation may be represented in matrix format as $\tilde{R} = (\tilde{u}_{ik})_{n\times m}$. The expert's degree of confidence in evaluating each alternative over all attributes has been considered as above.

3.2.4. Computation of average agreement degree of each expertusing OWA operator

The ordered weighted aggregation operator [31]is incorporated to address higher weight to high similarity between two experts. Thus obtainedagreement degree$\tilde{A}(J_{ik})$ of each expert J_i against each alternative gives a more meaningful definition as follows:

$$A(J_{ik}) = \frac{1}{n-1} \sum_{\substack{t=1 \\ t\neq i}}^{n} \tilde{S}_{itk} \,(\times) \omega_t for k = 12, \dots m \qquad (2)$$

Where $\tilde{S}_{itk} = \begin{cases} \tilde{S}(\tilde{u}_{ik}, \tilde{u}_{tk}) & for\ i \neq t \\ 1 & for\ i = t \end{cases}$

Suppose$\varpi = (\varpi_1, \varpi_2, \dots \varpi_{n-1})$ are weights of OWA operator and $\omega_t = \varpi_t if t < i$ and $\omega_t = \varpi_{t-1} if t > i$.

Here $S_{itk}$is the agreement between the opinions of the experts $J_i$and J_t is the similarity value between the generalized fuzzy numbers $\tilde{u}_{ik}$ and $\tilde{u}_{tk}$ and measured by employing the fuzzy similarity measure [25]. In the next section, an iterative process is developed with the help of the matrices mentioned above, to reach a group consensus opinion.

3.3 An iterative process

The steps of the proposed fuzzy iterative process have been demonstrated as follows. The aggregation procedure helps to combine the fuzzy opinions of each expert into a fuzzy number, denoted as $\widetilde{GR}_k(\,k = 1,2, \dots m)$, for representing the common opinion of these experts.

Step 1: Set l as the iteration number.$\widetilde{N}_i^{(l)} = \widetilde{N}_i, i = 1,2, \dots n$and $\widetilde{N}^{(l)} = \widetilde{N}$.
Step 2: Compute$\tilde{R}^{(l)} = (\tilde{u}_{ik}^{(l)})_{n\times m}$.
Step 3: Compute the average agreement degree$\tilde{A}(J_{ik}^{(l)})$using OWA operator and the corresponding degree of confidence $w(J_{ik}^{(l)})$ of each expert. If all $\tilde{A}(J_{ik}^{(l)}) \geq \tilde{\delta}$($\tilde{\delta}$is the threshold representing the acceptable level of the average agreement of all the experts through consensus) go to Step 5. Otherwise go to the next step 2.

Step 4: Let $\widetilde{N}_i^{(l+1)} = (\tilde{r}_{ijk}^{(l)})_{p\times m}$ and $\widetilde{N}_i = (\tilde{r}_{jk}^{(l+1)})_{p\times m}$

Where $\tilde{r}_{ijk}^{(l+1)} = c\left(\tilde{r}_{ijk}^{(l)}\right) + (1-c)(\tilde{r}_{jk}^{(l)})$ and

$$\tilde{r}_{jk}^{(l+1)} = \sum_{i=1}^{n} \tilde{r}_{ijk}^{(l+1)} \quad for\ j = 1,2 \dots p;\ k = 1,2, \dots m\ and\ i = 1,2, \dots n.$$

Here c has been introduced to assess the relative importance of the normalized decision matrix for individual. Then make $l = l + 1$, go to Step 2.

Step 5: Output l and $\tilde{A}(J_{ik}^{(l)})$for each expert J_i against each alternative A_k(using Eq. (2)). Presently the average agreement degree of individual expert with others is within acceptable range. So the relative agreement degree (RAD) of each expert J_i on each alternative$A_k (k = 1,2 \dots m)$is as follows:

$$\widetilde{Rad}_{ik} = \frac{\tilde{A}(J_{ik}^{(l)})}{\sum_{i=1}^{n} \tilde{A}(J_{ik}^{(l)})} \quad (3)$$

where the corresponding degree of confidence of the expert regarding $\widetilde{Rad}_{ik}$ is: $w(\widetilde{Rad}_{ik}) = \min_{1 \le i \le n} w(J_{ik}^{(l)})$.

Step 6: Finally, for each alternativeA_k, calculate the overall fuzzy numbercombining all experts' opinions $\widetilde{GR}_k$as:

$$\widetilde{GR}_k = \sum_{i=1}^{n} \widetilde{Rad}_{ik} (\times) \tilde{u}_{ik}^{(l)} \quad (4)$$

where the confidence level of the experts' overall opinion about each alternative $A_k (k = 1,2 \dots m)$ is:$w_k = \min_{1 \le i \le n} (w(\widetilde{Rad}_{ik}), w_{ik}^{(l)})$.

Step 7: End.

3.4. Approximate Reasoning Approach: Ranking Procedure

After aggregation of individual expert's judgments into a group consensus opinion, the goal of this stage is the determination of the best preference among the alternatives. Suppose, in order to evaluate the alternatives for different attributes the decision makers use the linguistic variables which are expressed in terms of GTFNs as shown in Table 1.

Table 1. Linguistic variables for the ratings of alternatives

Linguistic variables	GTFNs
Very Poor (VP)	(0; 0, 1)
Poor (P)	(1; 1, 2)
Fair (F)	(3; 2, 2)
Medium Good (MG)	(5; 2, 2)
Good (G)	(7; 2, 2)
Very Good (VG)	(9; 2, 1)
Excellent (Ex)	(10; 1, 0)

Then the similarity between the overall fuzzy score $\widetilde{GR}_k$ of the kth alternative and each term given in Table 1 has been computed. The higher value of the similarity measure indicates higher matching.

Suppose, $\widetilde{GR}_1$ and$\widetilde{GR}_2$ be the aggregated fuzzy scores of the alternatives A_1 andA_2, respectively and w_1, w_2 be the corresponding confidence level of the experts' overall opinions. Now the ordering has been done as follows:

- ✓ First A_1 and A_2 are labeled with the terms given in Table 1 where the similarity values are maximum.

- ✓ If the labels are different, then the ordering is either$A_1 > A_2$ or $A_1 < A_2$ according to the label given in Table 1.

- ✓ If A_1 and A_2 both are labeled with the same term, say $\mathfrak{I}^*$, then
  (i) $A_1 > A_2$ for $\tilde{S}\left(\widetilde{GR}_1, \mathfrak{I}^*\right) \geq \tilde{S}\left(\widetilde{GR}_2, \mathfrak{I}^*\right)$

  (ii) $A_1 < A_2$ for $\tilde{S}\left(\widetilde{GR}_1, \mathfrak{I}^*\right) \leq \tilde{S}\left(\widetilde{GR}_2, \mathfrak{I}^*\right)$

If $\tilde{S}\left(\widetilde{GR}_1, \mathfrak{I}^*\right) = \tilde{S}\left(\widetilde{GR}_2, \mathfrak{I}^*\right)$, then for ranking the alternatives, the attention has been given to the confidence level of the experts' overall opinions about an alternative and the ranking may be done as follows:

$A_1 > A_2 for\ w_1 > w_2.$
$A_1 < A_2 for\ w_1 < w_2.$

In the ranking process described above, it is clear that after calculating the similarity measure, a GFN is obtained. While comparing two fuzzy similarity measures, i.e., two GFNs, the ranking process proposed by Wang and Lee has been applied [26].

3.5. Empirical Case of Assessing an Optimal Diagnostic Laboratory

A five experts, say J_1, J_2, J_3, J_4 and J_5 committee is formed to choose the best diagnostic laboratory of a certain locality. The number of existing diagnostic laboratories considered in the comparison is five and denoted as $A = \{A_1, A_2, A_3, A_4, A_5\}$. The attributes are described properly using linguistic variables and can be transformed to associated fuzzy numbers. The decision makers employ the linguistic variables shown in Table 2 to evaluate the laboratories against each of the benefit attributes $(C_1 - C_5)$. The decision makers employ the linguistic variables shown in Table 3 to evaluate the laboratories against each of the cost attributes $(C_6 - C_7)$.

Table 2: Linguistic variables for ratings of alternatives with respect to benefit attribute

Linguistic variables	GTFNs
Very Poor (VP)	(0; 0, 1)
Poor (P)	(1; 1, 2)
Fair (F)	(3; 2, 2)
Medium Good (MG)	(5; 2, 2)
Good (G)	(7; 2, 2)
Very Good (VG)	(9; 2, 1)
Excellent (Ex)	(10; 1, 0)

Table 3: Linguistic variables for the ratings of alternatives with respect to cost attribute

Linguistic variables	GTFNs
Extreme Low (VL)/ Extreme Fast (EF)	(1; 1, 1)
Very Low (VL)/ Very Fast (VF)	(2; 1, 1)
Low (L)/ Fast (F)	(3; 1, 1)
Medium Low (ML)/ Medium Fast (MF)	(4; 1, 1)
Satisfactory (S)/ Normal (N)	(5; 1, 1)
Medium High (MH)/ Medium Slow (MS)	(6; 1, 1)
High (H)/ Slow(SL)	(7; 1, 1)

Expensive (E)/ Very Slow(VS) Very Expensive(VE)/ Extreme Slow (ES)	(8; 1, 1) (9; 1, 1)

Subsequently the ratings of laboratories given by the experts under all attributes are presented in Table 4, which is given in the next page.

Table 4: The ratings of alternative given by decision makes for all attributes

			Experts				
	Attributes	Alternatives	J_1	J_2	J_3	J_4	J_5
BENEFIT	C_1	A_1	MG	G	MG	MG	MG
		A_2	G	G	MG	G	MG
		A_3	VG	G	F	G	F
		A_4	F	F	F	F	F
		A_5	P	P	P	P	P
	C_2	A_1	G	MG	F	G	F
		A_2	VG	VG	VG	VG	VG
		A_3	MG	G	VG	MG	VG
		A_4	MG	MG	MG	MG	MG
		A_5	G	MG	G	G	G
	C_3	A_1	F	G	G	P	G
		A_2	VG	VG	G	VG	G
		A_3	G	MG	VG	G	VG
		A_4	VG	G	VG	VG	VG
		A_5	G	G	MG	MG	MG
	C_4	A_1	VG	G	VG	VG	G
		A_2	VG	VG	VG	VG	VG
		A_3	G	VG	MG	G	MG
		A_4	MG	F	MG	MG	MG
		A_5	F	MG	F	F	F
	C_5	A_1	F	MG	F	F	F
		A_2	VG	MG	G	G	MG
		A_3	G	F	MG	VG	MG
		A_4	F	P	F	F	F
		A_5	F	F	F	F	F
COST	C_6	A_1	MH	MH	MH	MH	MH
		A_2	MH	MH	MH	MH	MH
		A_3	E	E	E	E	E
		A_4	S	S	MH	S	MH
		A_5	MH	MH	MH	MH	MH
	C_7	A_1	N	MS	MS	MS	MS
		A_2	N	MS	N	N	N
		A_3	MF	MF	MF	MF	MF
		A_4	SL	F	SL	SL	SL
		A_5	SL	F	SL	SL	SL

The experts also use the linguistic variables from Table 5 [27] to assess the importance of different attributes and present it in Table 6.

Table 5: Linguistic variables for the importance of each attribute

Linguistic variables	GTFNs
Very Low (VL) Low (L) Medium Low (ML) Medium (M)	(0; 0, 0.1) (0.1; 0.1, 0.2) (0.3; 0.2, 0.2) (0.5; 0.2, 0.2)

Medium High (MH)	(0.7; 0.2, 0.2)
High (H)	(0.9; 0.2, 0.1)
Very High (VH)	(1.0; 0.1, 0)

Table 6: The importance of the attributes given by each expert

Attributes	Experts				
	J_1	J_2	J_3	J_4	J_5
C_1	H	VH	MH	MH	MH
C_2	VH	VH	VH	H	H
C_3	VH	H	H	H	H
C_4	VH	VH	VH	H	VH
C_5	M	MH	MH	MH	H
C_6	MH	H	H	H	VH
C_7	L	ML	ML	ML	L

For sake of simplicity and without any loss of generality, throughout this problem the degrees of confidence (w) of the experts in representing their opinions are considered as one, i.e. w=1. First the individual decision matrices $\widetilde{D}_i = \left(\tilde{a}_{ijk}\right)_{7\times5} (i = 1,2, \ldots 5)$ with the help of Table 4 are constructed. Then the proposed decision making technique can be applied step wise as follows:

Step 1: The normalization method is utilized to derive the normalized fuzzy decision matrix $\widetilde{N}_i = \left(\tilde{r}_{ijk}\right)_{7\times5}$ from the individual fuzzy decision matrix $\widetilde{D}_i = \left(\tilde{a}_{ijk}\right)_{7\times5}$, respectively. Then the aggregated matrix of first kind $\widetilde{N} = \left(\tilde{r}_{jk}\right)_{7\times5}$ is constructed from the normalized fuzzy decision matrices $\widetilde{N}_i = \left(\tilde{r}_{ijk}\right)_{7\times5}$ $(i = 1,2, \ldots 5)$.

Step 2: After that, Eq. (1) is employed to fuse the individual normalized fuzzy decision matrices $\widetilde{N}_i = \left(\tilde{r}_{ijk}\right)_{7\times5}$ $(i = 1,2, \ldots 5)$ into aggregated matrix of second kind $\widetilde{R} = (\tilde{u}_{ik})_{5\times5}$ $\tilde{R} = (\tilde{u}_{ik})_{5\times5}$. The result is presented in Table 7.

Table 7: The evaluation of each expert for every alternative over all attributes

Evaluation of the alternatives	Alternatives				
	A_1	A_2	A_3	A_4	A_5
$\tilde{u}_{1k}$	(0.583;0.335,0.375)	(0.829;0.414,0.397)	(0.670;0.370,0.409)	(0.562;0.330,0.373)	(0.472;0.284,0.354)
$\tilde{u}_{2k}$	(0.655;0.342, 0.40)	(0.789;0.373,0.378)	(0.640;0.327, 0.368)	(0.479;0.30, 0.393)	(0.489;0.297,0.392)
$\tilde{u}_{3k}$	(0.628;0.379,0.814)	(0.798;0.434, 0.481)	(0.668;0.389,0.431)	(0.583;0.361,0.414)	(0.482;0.309,0.391)
$\tilde{u}_{4k}$	(0.623;0.410,0.476)	(0.822;0.507, 0.518)	(0.670;0.438, 0.519)	(0.564;0.384,0.452)	(0.447;0.321,0.407)
$\tilde{u}_{5k}$	(0.610;0.358,0.437)	(0.794;0.420, 0.460)	(0.682;0.385,0.427)	(0.600;0.355,0.411)	(0.496;0.307,0.392)

Step 3: Utilizing Eq. (2), the ordered weighted average (OWA) [31] agreement degree of each expert J_i for every alternative A_k is calculated and presented with weight vector [29,30] $\overline{\omega} = (0.475,0.325,0.175,0.025)$ for `*orness(w)*=0.75' in Table 8.

Table 8: The average agreement degree of each expert J_i against each alternative

Average agreement degree	Alternatives A_1	A_2	A_3	A_4	A_5
$\tilde{A}(J_{1k})$	(0.964;0425,0.036)	(0.979;0.414,0.021)	(0.996;0.41,0.004)	(0.983;0.38, 0.017)	(0.985;0.338,0.015)
$\tilde{A}(J_{1k})$	(0.967;0.392,0.033)	(0.988;0.424,0.012)	(0.971;0.401,0.029)	(0.908;0.353,091)	(0.99;0.344,0.01)
$\tilde{A}(J_{1k})$	(0.986;0.564,0.014)	(0.99;0.445,0.01)	(0.995;0.444,0.005)	(0.979;0.387,0.017)	(0.99;0.347,0.01)
$\tilde{A}(J_{1k})$	(0.987;0.507,0.013)	(0.983;0.463,0.016)	(0.996;0.435,0.004)	(0.984;0.394,0.016)	(0.968;0.347,0.032)
$\tilde{A}(J_{1k})$	(0.982;0.466,0.018)	(0.991;0.442,0.01)	(0.987;0.426,0.013)	(0.97;0.386, 0.029)	(0.987;0.346,0.013)

The standard fuzzy scale that contains the entire spectrum of the experts' fuzzy threshold is considered as shown below. Therefore, the average agreement degree of individual expert should reach at least $\tilde{\delta} = (0.7; 0.2,0.3)$. After calculating the average agreement degree of each expert against every alternative, it is observed from Table 8 that all $\tilde{A}(J_{ik}) \geq \tilde{\delta}\ for\ i = 1,2 \dots 5\ and\ k = 1,2 \dots 5$.

Here the threshold value $\tilde{\delta}$ has been set by the experts based on their subjective evaluation of the particular situation under consideration. So it may be argued that this threshold value is more suitably modeled by a fuzzy number rather than by a crisp one.For example, expert's fuzzy threshold $\tilde{\delta}$ may take any value on the standard fuzzy scale having five polar terms - almost not, considerable, moderate, well, fully satisfied [28]. The following set of fuzzy numbers are considered in this regard,

Almost not=(0; 0,.3); Considerable= (0; .2, .5); Moderate=(.2; .5,.7); Well=(.5; .7,1); Fully satisfied=(.7; 1,1)

Step 4: Therefore, the RAD of each expert $J_i (i = 1,2 \dots 5)$ against each alternative $A_k (k = 1,2 \dots 5)$ is calculated using Eq. (3) and given in Table 9.

Table 9: The relative agreement degree of each expert J_i against each alternative A_k

RAD	Alternatives				
	A_1	A_2	A_3	A_4	A_5
$\widetilde{Rad}_{1k}$	(0.197;0.092,0.102)	(0.199;0.087,0.092)	(0.201;0.085,0.087)	(0.204;0.086,0.084)	(0.20;0.072,0.073)
$\widetilde{Rad}_{2k}$	(0.198;0.085,0.102)	(0.2;0.089,0.091)	(0.196;0.083,0.09)	(0.188;0.08,0.093)	(0.20;0.073,0.072)
$\widetilde{Rad}_{3k}$	(0.202;0.12,0.10)	(0.201;0.093,0.091)	(0.201;0.092,0.087)	(0.203;0.087,0.084)	(0.201;0.074,0.073)
$\widetilde{Rad}_{4k}$	(0.202;0.109,0.1)	(0.199;0.097,0.092)	(0.201;0.09,0.087)	(0.204;0.09,0.084)	(0.197;0.074,0.075)
$\widetilde{Rad}_{5k}$	(0.201;0.1,0.1)	(0.201;0.092,0.091)	(0.20;0.088,0.088)	(0.201;0.087,0.085)	(0.21;0.074,0.073)

Step 5: Finally, using Eq. (4) the overall fuzzy numbers combining all experts' opinions are calculated and given in Table 10.

Table 10: The final evaluation combining all experts' opinions for each alternative A_k

Alternatives	The final aggregation results
A_1	$\widetilde{GA}_1$= (0.620; 0.679, 0.814)
A_2	$\widetilde{GA}_2$ = (0.806; 0.799, 0.815)
A_3	$\widetilde{GA}_3$= (0.665; 0.673, 0.723)
A_4	$\widetilde{GA}_4$= (0.559; 0.587, 0.648)
A_5	$\widetilde{GA}_5$= (0.481; 0.481, 0565)

Applying the ranking method, mentioned in section 3.4, the ranking result is obtained as A_2, A_3, A_1, A_4, A_5. Therefore the best laboratory is$\boldsymbol{A_2}$.

4. CONCLUSIONS

In order to provide good services to the patients, the health-care department should identify good diagnostic laboratory. In view of this, the paper presents a scientific framework to assess the laboratories. In the process of decision making, several alternatives are considered and their performance are evaluated in terms of many different conflicting attributes, leading to a large set of linguistic data. During the evaluation process, the importance weights are selected in such a manner that the technical knowledge, cleanliness, and service of the laboratories are given more importance.This information may help the laboratories to enhance their performance in terms of the above-mentioned attributes. Moreover, this process may also help the health-care department to identify the critical performance criteria that should necessarily be followed while offering working license to a new diagnostic laboratory.

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