ERGODIC CAPACITY ANALYSIS FOR UNDERLAY COGNITIVE RADIO SYSTEM

Indu Bala[1], Manjit Singh Bhamrah[2] and Ghanshyam Singh[3]

[1,2]ECE Department, Punjabi University, Patiala, India
[3]ECE Department,JUIT, Solan, India

ABSTRACT

Cognitive radio technology has been proposed as a viable solution to the spectrum scarcity problem faced by world today. The technology allows opportunistic spectrum access to the licensed frequency band by unlicensed user without causing any harmful interference to the licensed primary user. In this paper, ergodic channel capacity is investigated for underlay spectrum sharing system under maximum and received power constraint at licensed primary receiver. The time varying discrete time fading channels are assumed to undergo Rayleigh flat fading environment. Numerical simulations have been done to support theoretical results.

KEYWORDS

Cognitive Radio, Channel State Information, Rayleigh Fading, Ergodic Capacity.

1.INTRODUCTION

Since, wireless products have become an integral part of modern lifestyle; the 21st century has witnessed the rapid deployment of wireless devices and applications in market. All these bandwidth hungry applications have increased the demand of electromagnetic spectrum. Traditionally, spectrum allocation policy is very inflexible in a sense that frequency band are exclusively licensed to the user for long term access with restriction on maximum transmission power to shield systems from mutual interference all the time. Since, most of the spectrum is already assigned, with the emergence of new wireless applications and constant need of mobile internet access, the demand for the wireless spectrum has increased manifolds and it is becoming extremely difficult to find vacant frequency band to deploy new wireless applications or to enhance the existing ones[1-2].

Cognitive radio technology has been proposed as a viable solution to the spectrum scarcity problem faced by world today. The technology allows opportunistic spectrum access to the licensed frequency band by unlicensed user without causing any harmful interference to the licensed primary user. The novel technology aims to have adaptive and flexible communication equipments designed in a way to monitor spectrum variations constantly over a wide range of frequencies and to adapt transmission parameters such as carrier frequency, bandwidth, modulation scheme, data rate and transmission power etc. this motivates the further investigation of cognitive radio network as a mean to mitigate spectrum scarcity problem. As a matter of fact, it has become hot topic of research and attracting researchers and academicians all over the world to propose new communication protocols to exploit the full capability of cognitive radios and to redefine the fundamental limits of the channel capacity for cognitive radios.

In this regard, a communication protocol is proposed in [3] to determine the channel conditions and to adapt its transmission signal to occupy the vacant band. Later, the approach is generalized to allow primary and secondary users to transmit simultaneously at the same time or at same frequency over additive white Gaussian noise (AWGN) channel and derived capacity bound for the same [4]. Most of the work previously has considered maximum transmit power constraint to limit the fundamental capacity of cognitive radio network, Received power constraint at primary receiver was firstly considered in [5] to explore the capacity of spectrum sharing channels. Author has used received power constraint on primary receiver. Later, ergodic capacity for point to point configuration is investigated either for maximum transmit power or average received power constraint at primary user receiver. It has been shown in [5] that significant capacity gain can be achieved if channel is varying due to fading. The same channel can be exploited by secondary user by transmitting opportunistically at higher power levels. This is in contrast to the systems with maximum transmission power constraint, where capacity degrades significantly because of fading.

In this paper, we have investigated the usefulness of various channel capacities, namely Ergodic (Shannon) capacity, where it is assumed that the channel transitions over all the fading states, for underlay cognitive radio system. The underlay cognitive radio system is the one in which the secondary cognitive user adapts its transmission based on the knowledge about channel gains between secondary and primary user. The maximum transmission power and averaged received power constraint at primary user's receiver jointly to estimate these channel capacities under Rayleigh fading environment.

2. System and Channel Model

In this paper, we have considered underlay spectrum sharing system in which a secondary user is allowed to use the licensed frequency band along with the licensed primary user, as long as the amount of interference inflicted at primary user is within predefined constraints on average and maximum values of transmission power. The spectrum sharing scenario and system model is shown in Fig. 1. It has been assumed that perfect channel state information (CSI) is known ahead of time to the secondary transmitter and receiver in advance.

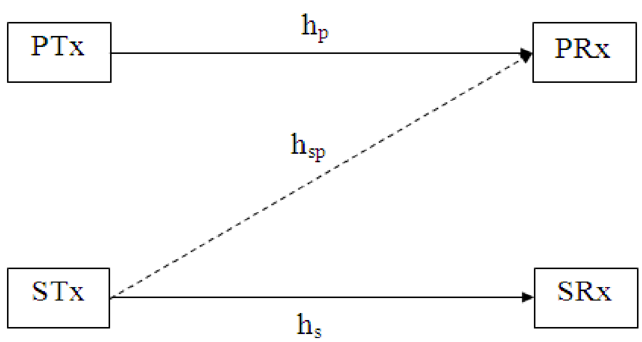

Fig. 1: Spectrum Sharing System Model

The time varying discrete-time fading channels are assumed to undergo Rayleigh flat fading, and the received signal power $y_s[n]$ at the secondary receiver depends on the transmitted signal power $x_s[n]$ according to

$$y_s[n] = h_s[n]x_s[n] + N_s \quad (1)$$

Where n is time index, $h_s[n]$ is the channel power gain between the secondary transmitter and receiver, and $N_s[n]$ represents AWGN. The power gain between the transmitter of secondary user and receiver of the primary user is $h_{sp}[n]$. Both channel power gains $h_s[n]$ and $h_{sp}[n]$ are independent and identically distributed (i.i.d.) exponentially distributed with unit mean. Furthermore, channel gains are assumed to be independent from the noise. The noise power spectral density and the signal bandwidth are represented by N_0 and Brespectively. Given that secondary transmission will not degrade signal quality at the primary receiver, constraints can be imposed on the received power at primary receiver. If $P_{maximum}$ is the maximum received power constraint and P_{avg} is the average power constraint, the corresponding constraint may be define as

$$P(h_s, h_{sp})h_{sp} \leq P_{maximum}, \quad \forall\, h_s, h_{sp} \tag{2}$$

$$\varepsilon_{h_s h_{sp}}\{P(h_s, h_{sp})h_{sp}\} \leq P_{avg} \tag{3}$$

where $P(h_s, h_{sp})$ represents the transmit power, and $\varepsilon_{h_s,h_{sp}}$represents the expectation over joint probability density function (PDF) of h_s and h_{sp}.

3. ERGODIC CAPACITY

In this section, the ergodic capacity for underlay spectrum sharing system is investigated for Rayleigh fading channel under received power constraint. The ergodic capacity is defined as maximum average rate with minimum probability of error under constrained transmission power. This is a useful performance metric for the systems in which the delay is not a constraint. Considering the average transmission power a constraint, system capacity under fading with full knowledge of CSI at secondary transmitter and receiver is derived in [6]. The closed form expression for the channel capacity of above system under Rayleigh fading is derived in [7]. Maximum and average power constraint on secondary transmission power was firstly considered in [8] for Gaussian channel and results were extended for quadrature Gaussian channel in [9]. In [10], the capacity of fading channel for maximum and average transmission power constraint was derived using multiplexed Gaussian codebook for optimum power allocation in time.

On contrary to the above, the constraint on received power is considered in [5] and the capacity of AWGN channels for single and multiple users is estimated. It was shown that the capacity of point- to- point AWGN channel with received power constraint is equal to the capacity of the channel under appropriately ranged transmit power constraint. The result, however, does not hold for fading channels. The ergodic capacity with average and maximum power constraint is given in [11] and optimal power allocation scheme has also been presented. The notations used in power calculations are given in Table 1.

S. No.	Symbol	Parameter
1	h_s	Channel gain between secondary Tx and Rx
2	h_p	Channel gain between Primary Tx and Rx
3	h_{sp}	Channel gain between Tx of SU and Rx of PU
4	$P_{maximum}$	Maximum allowed SU transmission power
5	P_{avg}	Average received power at Rx of PU
6	C_{er}	Ergodic Channel Capacity
7	N_0	Noise Variance
8	B	Channel Bandwidth
9	λ_0	Cut off Tthreshold value
10	γ_0	Signal to Noise Ratio

In this section, received power constraints are being investigated at a primary receiver under fading environment. These are the maximum and average received power constraints as given in (2) and (3) respectively. By using an approach similar to that used in [8], the channel capacity can be shown to be achieved through optimum utilization of the added power over time such that received power constraints are met. Therefore, the ergodic channel capacity in this case becomes an optimization problem and will be given as [12]:

$$\frac{C_{er}}{B} = \max_{P(h_s,h_{sp})\geq 0} \varepsilon_{h_s,h_{sp}} \left\{\log\left(1 + \frac{P(h_s,h_{sp})h_s}{N_0 B}\right)\right\}$$

$$\text{s.t.} \quad \varepsilon_{h_s,h_{sp}}\{P(h_s,h_{sp})h_{sp}\} \leq P_{avg},$$

$$P(h_s,h_{sp})h_{sp} \leq P_{maximum}, \forall\, h_s, h_{sp} \qquad (4)$$

The optimization problem in (4), without any constraint on maximum received power, is considered in [13] and solved by using Lagrangian optimization. Considering the problem under both power constraints and adopting a similar approach, the solution for (4) can be obtained using Lagrangian optimization by defining

$$L(P(h_s,h_{sp}),\lambda_0) = \varepsilon_{h_s,h_{sp}}\left\{\log\left(1 + \frac{P(h_s,h_{sp})h_s}{N_0 B}\right)\right\} - \lambda_0\left(\varepsilon_{h_s,h_{sp}}\{P(h_s,h_{sp})h_{sp}\} - P_{avg}\right) \qquad (5)$$

Musavian and Aissa [12] have provided a closed form expression for ergodic capacity of a system under power constraints as follows:

$$\frac{C_{er}}{B} = -\log\left(1 - \frac{P_{maximum}}{N_0 B + \gamma_0}\right) + \frac{P_{maximum}}{P_{maximum} - N_0 B}\log\left(\frac{P_{maximum}}{N_0 B \gamma_0}(N_0 B + \gamma_0 - P_{maximum})\right) \qquad (6)$$

where, $\lambda_0 = \frac{N_0 B}{\gamma_0}$, represents the optimal threshold value for which the transmission is suspended.

4. SIMULATION RESULTS & DISCUSSION

Figure 1 and Fig. 2 are showing the ergodic capacity under Rayleigh fading channel and the optimum cut off threshold associated with this capacity respectively. Impact of the maximum received power on ergodic capacity is shown in fig.1 by plotting ergodic capacity in nats/s/Hz verses P_{avg} for different values of $\rho = \frac{P_{maximum}}{P_{avg}}$. For the comparison purpose, results are compared with a case when only one constraint on either maximum or average received power is considered. It is clear from fig. 1 that ergodic channel capacity degrades with decrease in average received power when maximum received power constraint is applied on the top of the average received power constraint. With an increase in the parameter ρ ergodic capacity start converging towards a case of no maximum received power constraint.

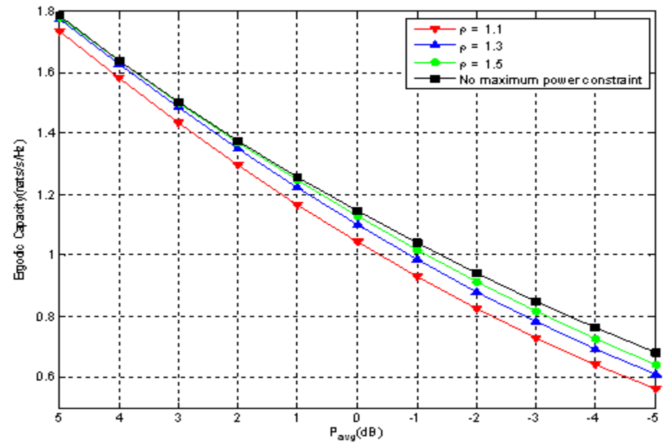

Fig.1: Ergodic capacity under maximum and received power constraint for different ρ.

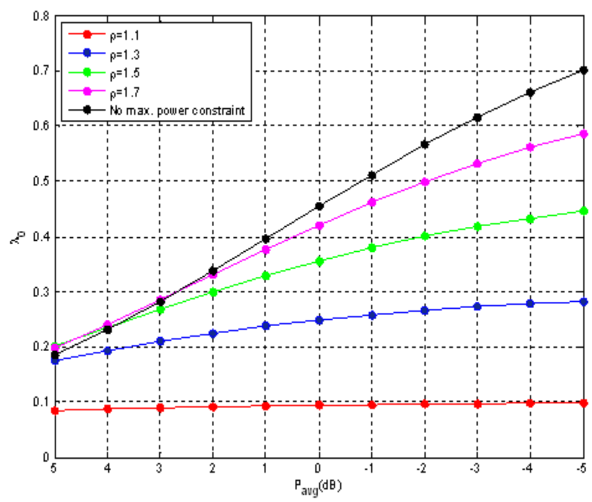

Fig. 2: Optimum cut-off threshold for different values of ρ.

Fig. 2 is showing the behaviour of optimum cut off threshold λ_0 under which transmission is suspended. It can be observed from the fig. 2 that optimum cutoff threshold value increases with an increase of the constraint on maximum received power at primary receiver. Moreover, for given constraint value, optimum threshold increases with decrease in the value of average received power at primary user's receiver.

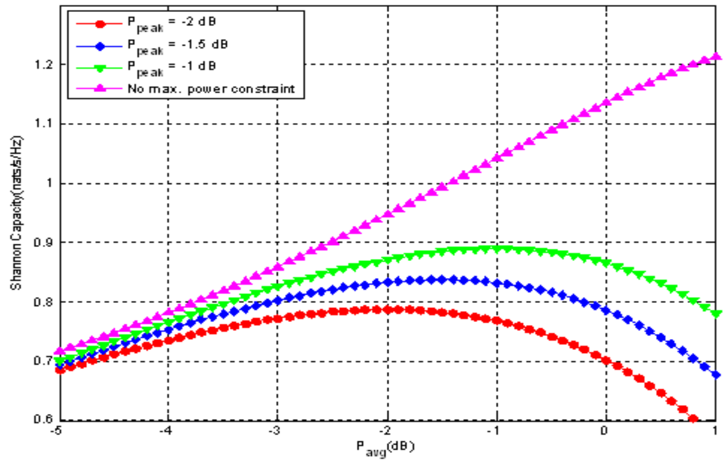

Fig.3: Ergodic Capacity under maximum and received power constraint for different $P_{maximum}$.

Fig. 3 is showing the behaviour of ergodic capacity with respect to the P_{avg} for the different values of maximum received power at primary receiver. It is clear that for the smaller values of average power, impact of maximum received power is negligible. For the given maximum power constraint, ergodic capacity increase with an increase in average received power and attain maximum value equals to the $P_{maximum}$ and then start decreasing.

5. CONCLUSION

The underlay spectrum sharing system is investigated for the ergodic capacity under maximum and received power constraint at licensed primary receiver for Rayleigh fading environment. It is shown that maximum received power constraint has almost negligible impact on ergodic capacity of channel when applied on the top of the average received power constraint. For the smaller values of average power, impact of maximum received power is negligible and ergodic capacity increases with an increase in average received power and attain maximum value equals to the $P_{maximum}$ and then start decreasing.

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